1978

Some aspects of statistically modeling the simulated plant-record method of life analysis

Karen Ann Hallaman Ponder
Iowa State University

Recommended Citation

Ponder, Karen Ann Hallaman, "Some aspects of statistically modeling the simulated plant-record method of life analysis " (1978). *Retrospective Theses and Dissertations*. 6412.
https://lib.dr.iastate.edu/rtd/6412

INFORMATION TO USERS

This was produced from a copy of a document sent to us for microfilming. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the material submitted.

The following explanation of techniques is provided to help you understand markings or notations which may appear on this reproduction.

1. The sign or "target" for pages apparently lacking from the document photographed is "Missing Page(s)". If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting through an image and duplicating adjacent pages to assure you of complete continuity.
2. When an image on the film is obliterated with a round black mark it is an indication that the film inspector noticed either blurred copy because of movement during exposure, or duplicate copy. Unless we meant to delete copyrighted materials that should not have been filmed, you will find a good image of the page in the adjacent frame.
3. When a map, drawing or chart, etc., is part of the material being photographed the photographer has followed a definite method in "sectioning" the material. It is customary to begin filming at the upper left hand corner of a large sheet and to continue from left to right in equal sections with small overlaps. If necessary, sectioning is continued again—beginning below the first row and continuing on until complete.
4. For any illustrations that cannot be reproduced satisfactorily by xerography, photographic prints can be purchased at additional cost and tipped into your xerographic copy. Requests can be made to our Dissertations Customer Services Department.
5. Some pages in any document may have indistinct print. In all cases we have filmed the best available copy.

University
Microfilms
International

300 N. ZEEB ROAD, ANN ARBOR, MI 48106
18 BEDFORD ROW, LONDON WC1R 4EJ, ENGLAND

7907275

PONDER, KAREN ANN HALLAMAN
SOME ASPECTS OF STATISTICALLY MODELING THE
SIMULATED PLANT-RECORD METHOD OF LIFE
ANALYSIS.

IOWA STATE UNIVERSITY, PH.D., 1978

University
Microfilms
International 300 N. ZEEB ROAD, ANN ARBOR, MI 48106

Some aspects of statistically modeling the simulated plant-record method of life analysis

by

Karen Ann Hallaman Ponder

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Department: Industrial Engineering
Major: Engineering Valuation

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

For the Major Department

Signature was redacted for privacy.

For the Graduate College

Iowa State University
Ames, Iowa

1978

TABLE OF CONTENTS

INTRODUCTION

Study of the mortality behavior of physical property emerged from the acceptance of the age-life relationship in depreciation estimates. For more than 200 years, statistical and actuarial methods have been used by insurance companies to analyze human mortality. The application of such techniques to determine service lives of physical property has become standard practice in regulated and unregulated industry as evidenced by a study made by the National Association of Railroad and Utility Commissioners (1943).

Depreciation Estimation

Depreciation calculations require estimates of the probable average service life of the property group or the probable average service life of the unit of property, and usually require an estimate of the probable retirement dispersion pattern of a property group. Winfrey (1967, p. 12) defined probable service life and probable average service life as:

> The probable service life of an individual unit is that period of time extending from its date of installation to the forecasted date when it will probably be retired from service.
>
> The probable average service life of a group of individual units is the average of the probable service lives of the units of the group.

Retirement dispersion pattern is determined from the distribution of the ages at retirement of the units comprising the property group.

Various systems have been devised to categorize retirement dispersions, but none has achieved the popularity of the Iowa type curves. The family of Iowa type curves represents a summary of studies of the survivor characteristics of many types of industrial and utility properties. The purpose of these studies was to generalize the attrition of units of physical properties into curves representing expected trends.

Winfrey (1967) developed 18 type curve shapes which are divided into three sets based on the position of the mode of the retirement frequency curve with respect to the average service life: six left modal, seven symmetrical, and five right modal. Couch (1957) categorized three curves whose mode is at or very near the origin and a straight line survivor curve. All four of these curves were designated origin modal. Subsequent research in industry has produced nine other type curves by combining the retirement frequency curves of the original Iowa type curves to form a new curve. There now exist 31 Iowa type curves.

Over forty years have passed since the Iowa type curves were compiled and published by Winfrey. In the past questions have been raised among the utility industry as to whether the Iowa type curves are still valid representations of mortality characteristics. A study by Russo (1978) repeated the same process of data collection and analysis performed by Winfrey. The clustering patterns Russo developed from the data gathered were found to be no better than the original Iowa type survivor curves.

The collection of data and subsequent statistical analysis are the fundamental tools for predicting service lives of physical property and retirement dispersion patterns. Edison Electric Institute (1952) cautions the depreciation analyst that the plant installed today or in general use may bear little or no resemblance to plant being retired or which has been retired. Hence, there is another important component of depreciation estimation which is described by Winfrey (1967, p. 9):

> While the author strongly recommends the development and use of the retirement data and and survivor curves as the basis of estimating the probable life of property units, he does not mean that the expert judgment should be done away with in favor of pure statistical treatment. Each individual item, each group of items, and each property or company must be dealt with in the light of its present condition, its character and amount of service production, and its relation to the present and probable future economic trends, art of manufacture, and management policies. Tables of probable service lives, type survivor curves, and statistical methods are simply means of recording past experience to use in predicting what the future service might be.

This investigation focuses on procedures used in life analysis which is the investigation of past experience. Life estimation seeks to predict future service lives based on informed judgment part of which is based on knowledge of past experience. The introduction of subjective factors which enter into judgment decisions is not within the empirical nature of this study.

Life Analysis

The techniques used in analyzing plant records to determine mortality characteristics from past experience can be categorized according to the type of data required. Actuarial data are data for which the property records contain the installation date for each retirement and each survivor. Mortality characteristics determined by actuarial methods are based on relationships of aged retirements.

The other category of property one encounters is semi-actuarial or unaged data. Semi-actuarial data contain records of the amount of property installed in each year, the amount of property retired each year, and the total plant balance or total survivors at all ages, with no knowledge of the age of a property unit at retirement. Semi-actuarial data is commonly found among property records for many reasons: records may have been started after the plant was installed; account classifications might have been changed within the life span of a property; property records may be nonexistent due to acquisition, sale, or merger; and for some classes of property it may be too difficult or too expensive to maintain complete records.

PRESENT METHODS OF ANALYZING SEMI-ACTUARIAL DATA

Semi-actuarial data are a severe handicap in specifying both the retirement dispersion and probable average life of a property. The techniques used in analyzing semi-actuarial data are the turnover method of life analysis and the simulated plant-record (SPR) method, both of which are described by Edison Electric Institute (1952).

Turnover Method of Life Analysis

The turnover method of life analysis requires a tabulation of the annual additions, retirements, and balances over a period of years approximating the average service life or more. The usual methods of handling the data are as follows:

1) Plot the cumulative retirements and the cumulative gross additions by years from the beginning of the account.
2) Accumulate annual retirements backwards from any given date until their sum equals the balance in the account at some earlier date. The period between the two dates is the "turnover period."
3) Accumulate gross additions backwards from any given date until their sum equals the balance in the account at any given date. The period necessary for this accumulation is the indicated "turnover period."

The turnover method of life analysis provides only an indication of average service life and does not yield an indication of retirement dispersion. Winfrey (1967, p. 35) advises caution in using this method:

> The average life determined will not be accurate unless the property has been continued in use at least one or two maximum life cycles, unless the replacements have about the same potential life expectancy as the retirements, and unless the property is maintained at about a constant number of service units. . . . For comparatively new properties, growing properties, and properties in which the potential lives of the units are changing rapidly, the turnover method is not to be recommended, or at least should not be used without due correction for these conditions.

These difficulties coupled with the lack of a retirement dispersion prompted the development of the simulated plant-record method.

Simulated Plant-Record Method

To overcome the limitations of the turnover method of life analysis, the simulated plant-record method of life analysis was introduced. The SPR method is the only procedure which yields estimates of <u>both</u> the probable average service life and retirement dispersion for semi-actuarial data.

To determine dispersion and average service life estimates for semi-actuarial data, the simulated plant-record method assumes a retirement distribution and average service life and that each year's additions are retired according to that pattern. If the property did indeed experience the assumed retirement pattern, then the resulting balances from the assumed retirement pattern would very nearly duplicate the actual balances of the account. The problem is to find a distribution which most nearly duplicates the actual plant balances. The criterion most commonly used to select the appropriate retirement distribution is to pick the one which minimizes the sum of squares differences between actual and simulated plant balances.

Hill (1922a,b) developed the basic principle of the SPR method more than 50 years ago as a procedure to analyze life experience of various classes of telephone plant. Hill's method provides solutions for average service life or dispersion when the other of these two parameters is known. It is indeterminant when solving for both parameters. Subsequent research has provided for simultaneous solutions of both parameters.

Basically, the SPR method is a trial and error procedure which attempts to duplicate the annual balances (or cumulative retirements) of a plant account by distributing the annual gross additions over time according to some assumed mortality distribution. Specifically, the dollars (or units) surviving at any date are estimated by multiplying each year's additions by the successive proportion surviving at each age obtained from the assumed mortality distribution. For a given year, the accumulation of survivors from each vintage estimates the actual plant balances for that year. This procedure is reiterated for different mortality distributions until a distribution is chosen that produces a set of simulated balances which most closely duplicates the actual balances. Most depreciation analysts use the criterion of producing a minimum sum of squares differences between the actual and simulated balances.

Bauhan (1947, 1948) developed the above procedure more than 30 years ago, and this method is the most widely used for analyzing semiactuarial data today. To aid in evaluating the selection of a representative distribution, Bauhan proposed the conformance index and the retirements experience index. The conformance index was devised to indicate the goodness of fit in relation to the size of the account and is defined as follows:

$$\text{Conformance index} = \frac{\begin{array}{c}\text{Average of Actual Balances in}\\ \text{Comparison Years}\end{array}}{\left\{\begin{array}{c}\text{Sum of squares differences between}\\ \text{actual versus simulated balances}\\ \text{in comparison years} \div \text{number of}\\ \text{comparison years}\end{array}\right\}^{\frac{1}{2}}}$$

Bauhan devised an arbitrary scale of comparison graded as: excellent for ratios over 75; good for ratios between 75 and 50; fair for ratios between 50 and 25; and poor for ratios between 25 and 0. The lack of empirical substantiation for this scale makes the application of the conformance index of doubtful validity.

Bauhan was concerned that the conformance index might be very high where there is little experience with the account. To correct this problem, he devised the retirements experience index which indicates the amount of experience with the account. This index is equal to the percentage of accumulated retirements of the first year's additions at that age representing the age of the account. The retirements experience index is graded on an arbitrary scale as follows: excellent for indexes over 75%; good for indexes between 75% and 50%; fair for indexes between 50% and 33%; poor for indexes between 33% and 17%; and valueless for indexes below 17%. Like the conformance index, the retirements experience index is an arbitrary measure with no empirical substantiation.

Another proposed criterion was devised by White and Cowles (1972). The index of variation is defined as:

$$\text{Index of variation} = 1000 \frac{\left[\dfrac{\text{Sum of squared differences from actual balances}}{\text{Number of test years}}\right]^{\frac{1}{2}}}{\text{Average actual balance}}$$

The smaller the index of variation the better would be the fit of the simulated balances to actual balance. This index of variation is equal to the reciprocal of the conformance index times 1,000. No scale to judge the quality of fit has yet been devised for the index of variation.

Variations of the simulated-plant record method have been developed by Whiton (1947) and Garland (1967). Whiton proposed the comparison of indicated retirements in place of the balances. At any date, cumulative retirements may be computed as the sum of the gross additions less the balance at that date. Both this indicated retirements approach and the balances method will select the same distribution because the magnitude of sum of squares deviations is the same for both methods. Whiton indicates the advantage of the indicated retirements approach is that for a given year the ratio of the sum of squares deviation to the cumulative retirements is greater than the ratio of the deviation to the plant balance. This, in turn, magnifies the deviation and gives a better indication of the goodness of fit.

Garland has approached the use of the retirements instead of balances in the period retirements method. This method compares actual versus simulated retirements occurring in a given time period. Specifically, the retirements occurring in a given year are computed as the difference between the beginning and ending balances plus the additions for that year. The advantage of this method according to Garland is that mortality characteristics being experienced in a recent time period are highly indicative of future retirement activity.

In the above methods, there are two underlying assumptions common to all these approaches. The first assumption is that the mortality experience of a given vintage is independent of the mortality experience of all other vintages. In other words, the occurrence of retirements from one vintage in no way affects the occurrence of retirements from any other vintage. Secondly, it is assumed that each vintage is a sample from the homogeneous population and is, hence, retired according to the same mortality distribution. Imposing these assumptions causes the SPR method to fail to detect shifts in dispersion and/or average service life between vintages.

R. E. White (1968) statistically modeled the SPR balances method and derived a test procedure in an attempt to eliminate the subjective judgment one finds in the arbitrary scales of the conformance index and retirements experience index. His method was applied by Rose (1972) and Rippe (1969) to real world data with little success. The chi-square statistic used was found to reject all but the most regular accounts.

A STATISTICAL MODEL OF THE SIMULATED PLANT RECORD BALANCES METHOD

The following statistical theory was developed by White (1968) to describe the SPR balances method.

Derivation of Survivor Curve

Let K be a discrete random variable representing the life of a unit of property where K = {1, 2, ..., m} and m is the finite maximum life. The probability density function (PDF) of K is the probability that the unit of property is retired at age k or

$$f(k) = P(K = k) = \pi_k \quad . \tag{1}$$

The cumulative distribution function (CDF) of K is the probability that the unit of property is retired before age k+1 or

$$f(k) = P(K \leq k) = \sum_{y=1}^{k} f(y) \quad . \tag{2}$$

A survivor curve of K may be defined as the probability that the unit of property survives through age k or

$$\theta_0 = 1$$

$$\theta_k = P(K > k) = 1 - \sum_{y=1}^{k} f(y) = \sum_{y=k+1}^{m} f(y) \tag{3}$$

$$(k = 1, 2, \ldots, m)$$

$\theta_{m+1} = 0$.

Derivation of the Balances Method Model

To relate the survivor curve notation derived above to the balances method of the SPR, the following symbols are defined below to represent property account activity:

T = a set of points in time with the same units of measure as K, where $T = (0, 1, 2, \ldots)$.

j, k = points in time where $j, k \varepsilon T$.

N_j = the number of units installed as a group at time j.

s_{jk} = the proportion of units installed at time j that are surviving at time k.

p_{jk} = the proportion of units installed at time j that are retired at time k.

$N_j s_{jk}$ = the number of units installed at time j that are surviving at time k.

$N_j \theta_{jk}$ = the number of units installed at time j expected to be surviving at time k, where $\theta_{jk} = \theta_{k-j}$.

$N_j p_{jk}$ = number of units installed at time j that are retired at time k.

$N_j \pi_{jk}$ = number of units installed at time j that are expected to be retired at time k, where $\pi_{jk} = \pi_{k-j}$.

B_k = total plant in service at time k or $\sum_{j=1}^{k} N_j s_{jk}$ where $j \leq k$.

D_k = total plant which has been retired from time k-1 to k = $\sum_{j=1}^{k} N_j p_{jk}$.

As shown in Figure 1, the retirements for a given property account are a composite of retirements from prior vintages, each vintage successively displaced by one unit of time. In general, D_k is equal to the sum of $N_1 p_{1,k}$, $N_2 p_{2,k}$, ..., $N_k p_{k,k}$. The total plant in service at any time, B_k, is equal to the sum of the additions through time k less the retirements through time k, or

$$\begin{aligned} B_k &= \sum_{i=1}^{k} N_i - \sum_{i=1}^{k} D_i = \sum_{i=1}^{k} N_i - \sum_{i=1}^{k} N_i p_{ik} \\ &= \sum_{i=1}^{k} N_i s_{ik} \qquad (i \leq k) \quad . \end{aligned} \tag{4}$$

Using the above relationships, the expectation of any balance or any retirement may be obtained.

$$E(D_k) = E\left(\sum_{j=1}^{k} N_j p_{jk} \right) = \sum_{j=1}^{k} N_j \pi_{jk} \tag{5}$$

$$\mu_k = E(B_k) = E\left(\sum_{j=1}^{k} N_j s_{jk} \right) = \sum_{j=1}^{k} N_j \theta_{jk} \tag{6}$$

Thus, while the random variable B_k denotes the actual book balance appearing in a property account, the expected value of B_k which is equal to μ_k denotes a simulated balance generated by the SPR balances method.

$$
\begin{array}{cc|ccccc|}
 & & 1901 & 1902 & 1903 & & 1930 \\
\hline
1901 & N_1 & P_{1,1} & P_{1,2} & P_{1,3} & \cdots & P_{1,30} \\
1902 & N_2 & & P_{2,1} & P_{2,2} & \cdots & P_{2,29} \\
1903 & N_3 & & & P_{3,1} & \cdots & P_{3,28} \\
\vdots & \vdots & & & & & \vdots \\
1930 & N_{30} & & & & & P_{30,1} \\
\hline
 & & D_1 & D_2 & D_3 & \cdots & D_{30}
\end{array}
$$

Figure 1. Example of property account notation relating additions and retirements

The N_j units retired at time j may be viewed as N_j independent trials where each trial can have one of several outcomes. The outcome of a particular unit or trial from N_j may be retirement at age 1, or at age 2, ..., or at age m. Thus the number retired at time k from the units installed at time j (N_jp_{jk}) may be described as a multinomial random variable. This distributional assumption coupled with the assumption that each vintage is subject to the same law of mortality enables derivations of variance and covariance structures for the retirements and the balances:

$$\begin{aligned} \text{var}(D_k) &= \text{var}\left(\sum_{j=1}^{k} N_j p_{jk} \right) \\ &= \sum_{j=1}^{k} \text{var}(N_j p_{jk}) \\ &= \sum_{j=1}^{k} N_{k+1-j} \pi_j (1 - \pi_j) \end{aligned} \tag{7}$$

$$\begin{aligned} \text{cov}(D_k\ D_{k'}) &= \text{cov}\left(\sum_{j=1}^{k} N_j p_{jk},\ \sum_{i=1}^{k} N_i p_{ik} \right) \\ &= - \sum_{i=1}^{k} N_{k+1-i} \pi_i \pi_{k'-k+i} \end{aligned} \tag{8}$$

$$\begin{aligned} \text{var}(B_k) &= \text{var}\left(\sum_{i=1}^{k} N_i - \sum_{i=1}^{k} D_i \right) \\ &= \text{var}\left(\sum_{i=1}^{k} D_i \right) \end{aligned}$$

$$= \sum_{i=1}^{k} \text{var } D_i + 2 \sum_{i=1}^{k} \sum_{j=i+1}^{k} \text{cov}(D_i, D_j)$$

$$= \sum_{i=1}^{k} N_{k-i+1} \left(1 - \sum_{j=1}^{i} \pi_j \right) \left(\sum_{j=1}^{i} \pi_j \right) \qquad (9)$$

$$\text{cov}(B_k \quad B_{k'},) = \text{cov} \left\{ \sum_{i=1}^{k} (N_i - D_i), \sum_{j=1}^{k} (N_j - D_j) \right\}$$

$$= \sum_{i=1}^{k} N_{k-i+1} \left(\sum_{j=1}^{i} \pi_j \right) \left(1 - \sum_{j=1}^{i+k'-k} \pi_j \right) \qquad (10)$$

Since the values for the additions are large (usually in dollar values) the $N_j p_{jk}$'s can be assumed to be normally distributed. Since each placement is independent of the others and the sum of independent normally distributed, then each balance B_k is normally distributed.

Using these assumptions, a mean vector and variance covariance matrix are given below

$$\underline{B} = \begin{bmatrix} B_1 \\ B_2 \\ \cdot \\ \cdot \\ \cdot \\ B_k \end{bmatrix} \qquad \underline{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \cdot \\ \cdot \\ \cdot \\ \mu_k \end{bmatrix}$$

$$\Sigma = E[(\underline{B} - \mu)'(\underline{B} - \underline{\mu})]$$

$$= \begin{bmatrix} \text{var } B_1 & \text{cov}(B_1,B_2) & \cdots & \text{cov}(B_1,B_k) \\ \text{cov}(B_1,B_2) & \text{var } B_2 & & \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ \text{cov }(B_1,B_k) & & & \text{var } B_k \end{bmatrix}$$

The vector of balances $\underline{B}$ has a multivariate normal distribution with mean μ and variance-covariance matrix Σ. Furthermore the quadratic form $z = (\underline{B} - \underline{\mu})\Sigma^{-1}(\underline{B} - \underline{\mu})'$ is a χ^2 variate with k degrees of freedom. Hence the null hypothesis that $\underline{B}$ did in fact come from a multivariate normal distribution with mean $\underline{\mu}$ and covariance matrix Σ may be tested by calculating the χ^2 statistic z.

White and Cowles (1970, p. 1207) formulated the null hypothesis to be tested in the following manner:

> The actual balances came from the parent population described by the distribution of simulated balances. Clearly, if it can be established with some level of certainty that the book balances did come from the same population as those of the simulated balances, then the mortality distribution used to derive the simulated balances can be accepted with the same level of certainty to be descriptive of the mortality characteristics of the account.

White (1968) met with limited success in attempts to apply the chi-square statistic to real and simulated data. For all but the most regular accounts, the chi-square statistic was too powerful and rejected the dispersion which produced the minimum sum of squares differences between actual and simulated balances. These difficulties prompted an investigation of tolerance regions.

Tolerance Regions

Chew (1966) defines a tolerance region as a region R which can be constructed such that the probability is γ that R contains at least (100P)% of the individuals in the population or such that the average or expected value of the proportion of the population contained in R is exactly (100P)%. This construction of a tolerance region is equivalent to finding a test function for the hypothesis testing problem. The tolerance region may take on different shapes. Two are examined in this research: the ellipsoidal shape and the rectangular parallelepiped shape.

By comparing results given by Chew (1966) and Fraser and Guttman (1956), is is evident that the chi-square statistic derived in the previous section is equivalent to the test of hypothesis given by the ellipsoidal shape. The rectangular parallelepiped shape is given below as derived by Chew (1966). Let

P = the proportion of the population in the tolerance region.

p = the number of comparison years or the number of dimensions in the population.

$z(\alpha)$ = upper 100α percent point of the standard normal distribution.

To form a rectangular parallelepiped region, the variance-covariance matrix for the balances must be diagonal, so a vector $\underline{y}$ must be found such that $\underline{y} = A\underline{B}$ and $\Sigma_y = A \, \Sigma_B \, A'$ is diagonal. Since Σ_B is a real symmetric matrix, A will be the matrix of eigenvectors of Σ_B, and the nonzero elements of Σ_y will be the eigenvalues of Σ_B. Thus the μ_i and σ_i values in formula (11) pertain to $\underline{y}$. The tolerance region formula for a rectangular parallelepiped region is:

$$\Pr\{\mu_i - z\left(\frac{1}{2} - \frac{1}{2} P^{1/p}\right) \sigma_i \le y_i \le \mu_i + z\left(\frac{1}{2} - \frac{1}{2} P^{1/p}\right) \sigma_i\} = P^{1/p} \qquad i = 1, 2, \ldots p \qquad (11)$$

Because the y's are uncorrelated, the probability that all p statements are true is P. By varying the values for P, it was hoped that the rectangular parallelepiped region would be less powerful than the chi-square statistic and allow acceptance of more accounts.

OBJECTIVES

The simulated plant record method has been the subject of great controversy in its application and analysis. The statistical model developed by White (1968) was an attempt to have a basis of comparison which was empirical in nature rather than subjective such as the conformance index or retirements experience index. To gain further understanding of statistically modeling the SPR method, this investigation has been undertaken.

The specific objectives of this study are as follows:

1. To examine the normality assumptions which underlie the statistical theory of the SPR method.
2. To determine the effect the shape of the tolerance regions used has on conclusions reached.
3. To examine modifications in statistical theory when the data are dependent on what occurred in previous years.
4. To develop a procedure by which data-dependent cases can be examined using independence assumptions.
5. To determine the effectiveness of the balances method in estimating the correct dispersion and service life.
6. To develop computer programs which aid in implementation of the objectives above.

This study is restricted to examination of the balances method. Since the balances method is the most popular approach used by depreciation analysts and it can be represented in a statistical manner, scrutiny will be given to this approach.

MONTE CARLO STUDY WITH CONSTANT ADDITIONS

In evaluating the effectiveness of a particular technique or methodology, the results must be compared against an acceptable standard. Real data are affected by many inputs only a few of which can be isolated. The importance of the inputs is usually impossible to ascertain. These inputs and their relative importance will vary with time as unforseen forces act upon them, making it impossible to observe the effect of any single factor held constant over time. The introduction of inflation into the analysis, for example, would cause accounts whose units are expressed in dollar values to have units of varying value, introducing even more complexity into the situation.

To aid in controlling the inputs of an account, a Monte Carlo study was undertaken to determine if the normality assumption of the balances upon which the statistical development rests is valid and what effects, if any, are involved when the shape of the tolerance region is changed. In generating the samples for the Monte Carlo study, the PGM program documented by Erbe (1971) was used.

Description of PGM Computer Program

The PGM is capable of simulating the life of a property account over a period of years. This program provides the option of either an expected value or a random value simulation. In the expected value simulation, the age frequency distribution of simulated retirements from each vintage will conform exactly to the smooth retirement frequency curve of the specified population. In the random value, an age frequency distribution will be produced that deviates about the expected values of a smooth retirement frequency curve.

Input variables include an average price per unit installed and a variable range above and below the average price. In simulating the retirement experience of an account, the initial placement and a desired growth rate are specified. The annual placement in succedding years is computed to maintain a specific growth in the plant balance. For each simulation year, the effective growth rate is sampled from a normal distribution with a mean of the specified growth rate and a standard deviation of 10% of the stated mean.

The decision rule used by the PGM to determine the annual placement in succeeding years is intricate. Given the initial placement, all the units placed are retired through age m, where m is the maximum age. In the second year, the placement is equal to the price per unit times the units that will retire in the second year from the initial placement divided by the percent surviving at the end of the first year taken from the assumed mortality distribution. Then all those units placed at the beginning of the second year are retired through age m. To determine the third year's placements, the price per unit is multiplied by the units which will be retired during the third year divided by the percent surviving at the end of the first year taken from the assumed retirement distribution. The units which will retire in the third year used in the above computation are made up of retirements from placements in years one and two. In general, the placement in year X is equal to the price per unit times the units which are computed to retire in year X divided by the percent surviving at the end of the first year. This decision rule has implications which will be discussed in connection with a data-dependent Monte Carlo study.

For the special case where a constant number of additions is added each year, there is a special option register which overrides the computation of additions to meet a specific balances but computes retirements as before. There is also an inflation parameter which can be used to reflect an annual price escalation.

The PGM provides the option of selecting a parent population from the original 18 Iowa type mortality curves developed by Winfrey (1967). The dispersion and average service life are assumed effective for all the time periods within the specified time span. The beginning vintage and the last vintage of the time span must be specified.

Test of Normality for Balances

To test the distribution of the balances for the previously developed statistical theory, 100 random samples with constant additions were generated. Use of the constant infusions model assures that the assumption that the occurrence of retirements from one vintage in no way affects the occurrence of retirements from another vintage is valid.

Each of these samples came from an Iowa type R1 dispersion with a ten year average service life. Additions of $100 were made for each year of study. No price variation nor inflation was introduced. Ten years of experience were generated. A complete listing of these samples is found in the Appendix for samples 1 through 100 inclusive.

In Figures 2 and 3, the distribution of balances in years 9 and 10 is given. From visual examination of these histograms, it is not immediately apparent if the assumption of normality is justified. To validate this distribution assumption, the chi-square goodness of fit test was applied to years five through 10. Since the additions are the same for each sample, the mean and variance in any given year are the same from sample to sample. The following means and variances as given in Table 1 can be obtained from formulas (6) and (9) applied to an R1-10 curve with additions of $100 per year.

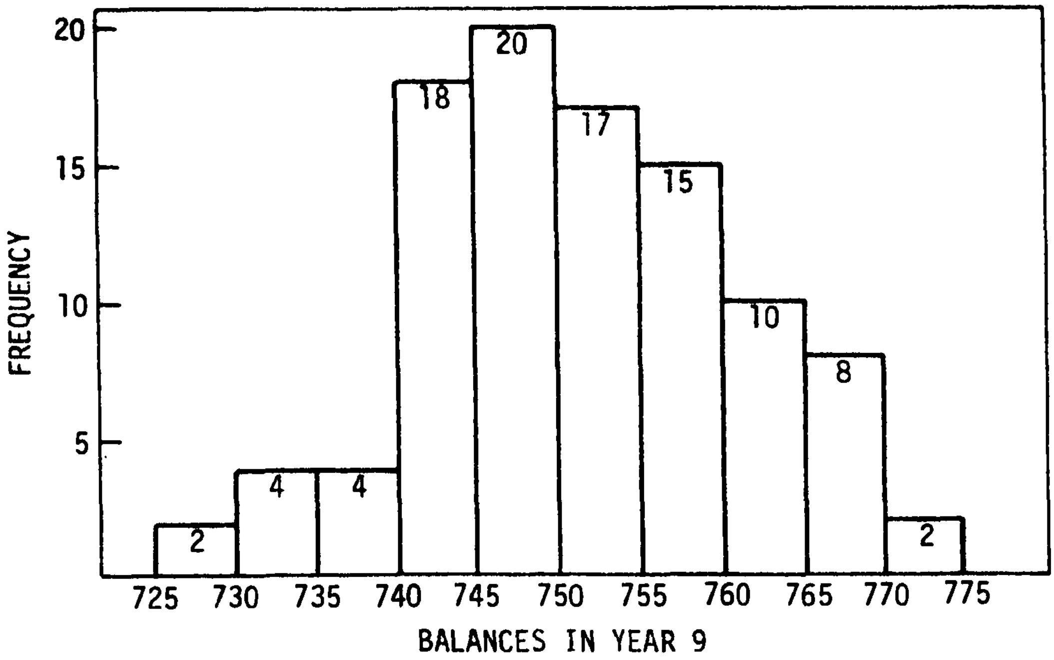

Fig. 2. Distribution of balances in year 9 taken from 100 samples of an R1-10

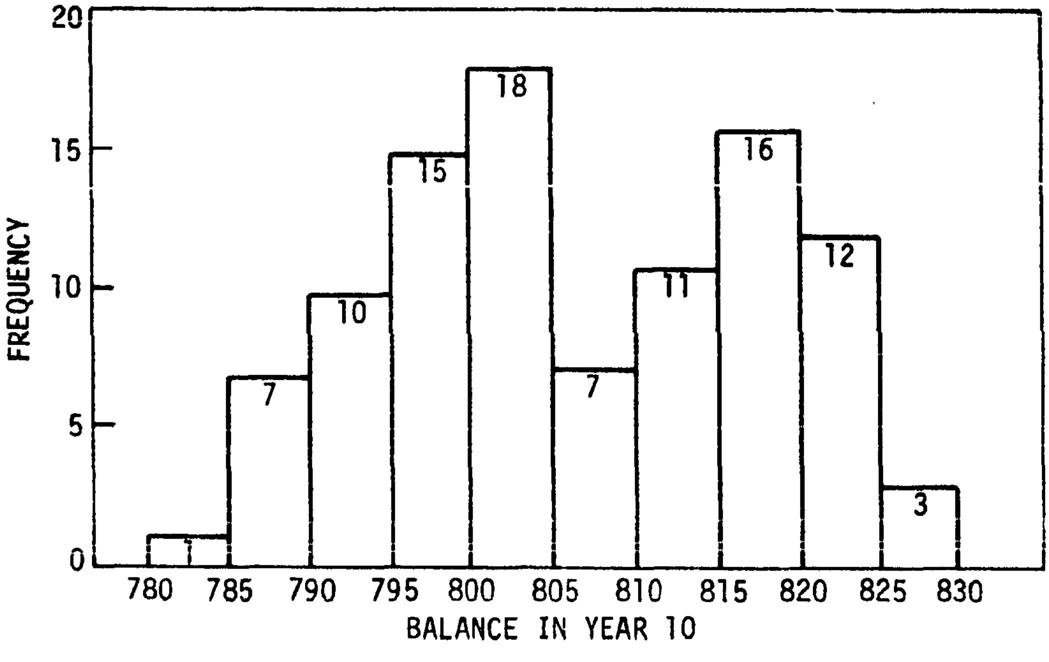

Fig. 3. Distribution of balances in year 10 taken from 100 samples of an R1-10

Table 1. Means and variances for balances for the case of constant infusions

Year	Mean	Variance
5	460.55	35.13
6	540.92	47.99
7	616.28	66.56
8	685.99	92.33
9	749.42	117.57
10	805.96	145.47

If the normality assumption is justified, a division of each year's balances in relation to how far each balance is from the standard deviation based upon the mean would reveal an observed value of how many values fall into each class. This can be compared to expected values if the normality assumption were correct. The chi-square goodness of fit test can then be applied:

$$\chi^2 = \sum_{\text{all classes}} \frac{(\text{Observed Frequency}-\text{Expected Frequency})^2}{\text{Expected Frequency}}$$

with degrees of freedom equal to the number of classes minus the number of parameters being estimated.

In Table 2, the data for years 5 through 10 are broken into different classes and the chi-square goodness of fit test value given. In this case five classes have been used and two

Table 2. Frequency of balances for years 5 through 10 falling in each category and the corresponding chi-square values for samples 1 through 100

Year	Frequency less than $\mu - 1.5\sigma$	Frequency between $\mu - 1.5\sigma$ and $\mu - .5\sigma$	Frequency between $\mu - .5\sigma$ and $\mu + .5\sigma$	Frequency between $\mu + .5\sigma$ and $\mu + 1.5\sigma$	Frequency greater than $\mu + 1.5\sigma$	Chi-square
Expected	6.68	24.17	38.30	24.17	6.68	
5	3	20	43	24	10	5.13
6	8	25	32	27	8	1.92
7	7	26	31	31	5	3.90
8	5	22	45	22	6	2.17
9	6	19	41	29	5	2.75
10	3	30	33	30	4	6.64

parameters, the mean and variance, are being estimated. Hence, these values are compared to $\chi^2_{3,.05} = 7.81$. For each year in question the chi-square values are less than 7.81, so at the 5% level of significance, the hypothesis of normality with the given mean and variance can be accepted.

The Effect of Tolerance Region Shape

Since the assumption of normality of the balances for the constant infusions case has been established, the question of how the shape of the tolerance region affects analysis will next be examined. In Figure 4, the bivariate distribution of the balances in years 9 and 10 is given. The balances in years 9 and 10 have the following mean and variance-covariance structure:

$$\underline{\mu} = \begin{bmatrix} 749.42 \\ 805.96 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} 117.57 & 105.42 \\ 105.42 & 145.47 \end{bmatrix}$$

Forming the chi-square statistic

$$Z = (\underline{B} - \underline{\mu}) \, \Sigma^{-1} \, (\underline{B} - \underline{\mu}) \tag{12}$$

for this example is equivalent to solving the equation:

$$(x - 749.42)^2(.024) - 2(.018)(x - 749.42)(y - 805.96)$$
$$+ (.019)(y - 805.96)^2 \leq \chi^2_{2,.05}$$

where x = balance in year 9 and y = balance in year 10.

Figure 5 shows the solution to the above equation. At the 5% level of significance, only one sample out of 100 was rejected.

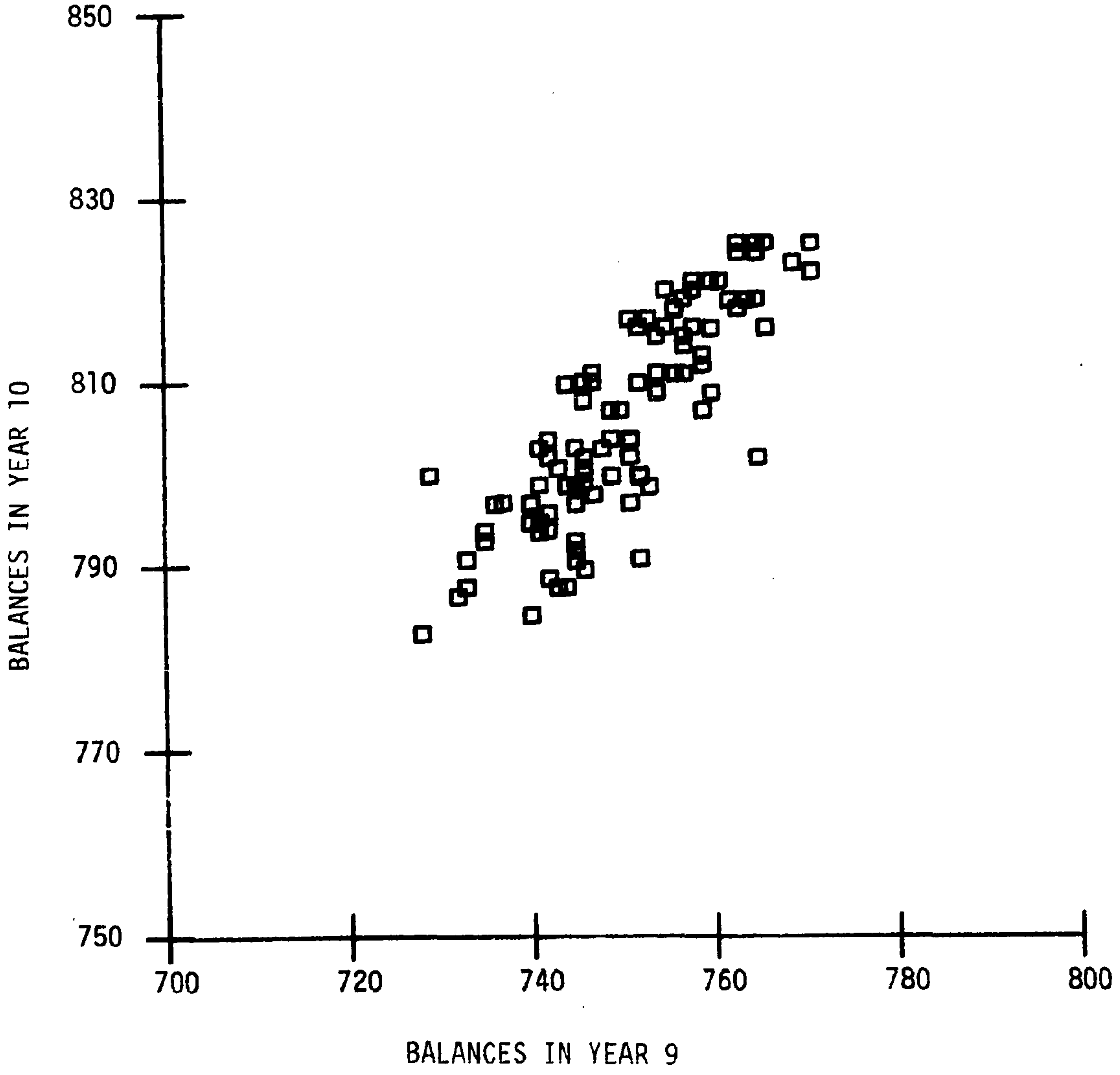

Fig. 4. Bivariate distribution of balances in years 9 and 10 for an R1-10

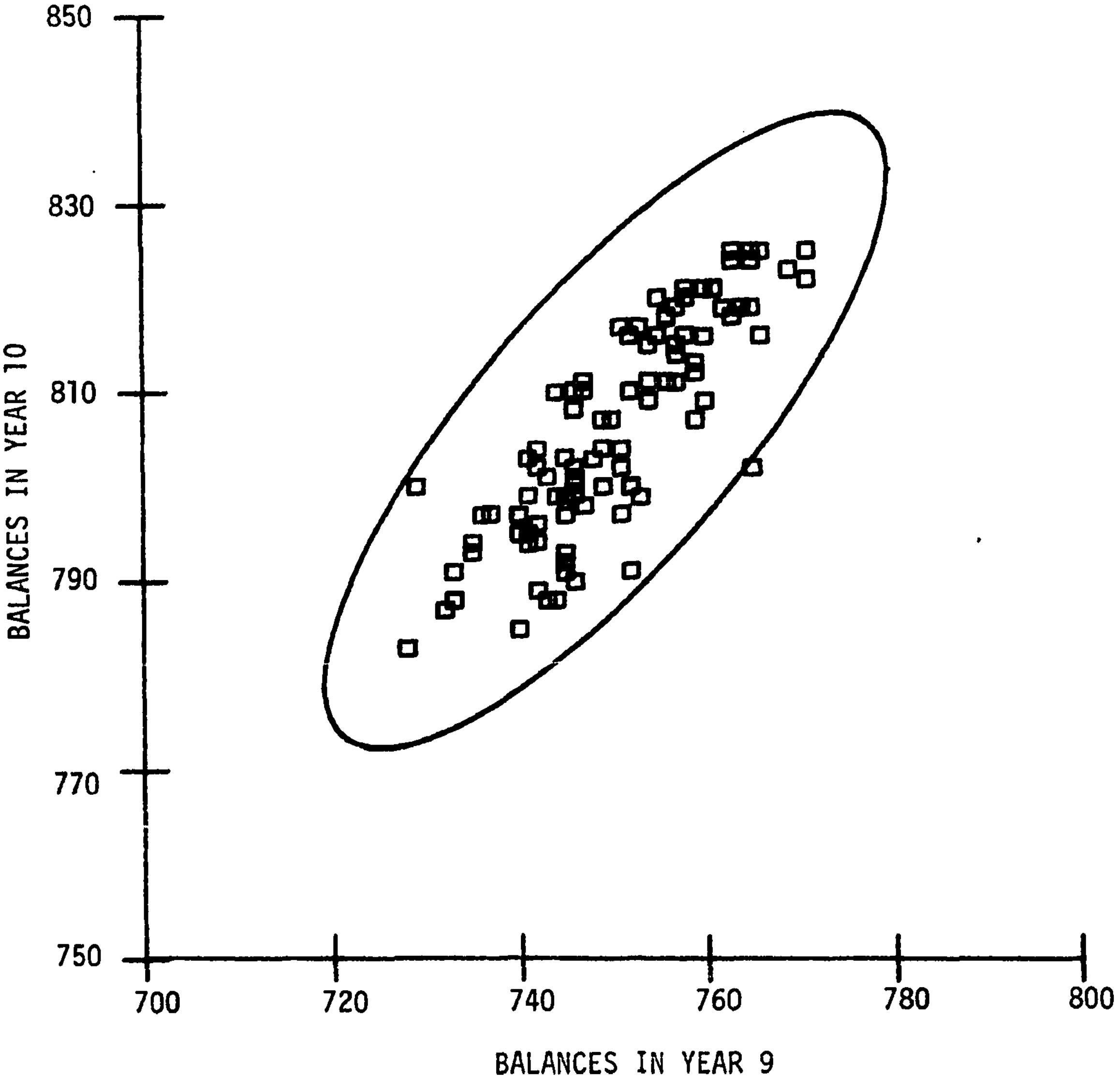

Fig. 5. Results of fitting ellipsoidal tolerance region to balances in years 9 and 10 from an R1-10

The rectangular parallelepiped region involves transformation of the actual balances by premultiplying times the eigenvectors of Σ. In this case,

$$x' = -.75x + .66y$$

$$y' = .66x + .75y$$

where x and y are defined as above. The graphical representation in Figure 6 can be compared to Figure 4 to visualize the change this transformation makes in the original data. This transformation changes the mean vector and variance-covariance structure to:

$$\mu' = \begin{bmatrix} -32.14 \\ 1100.07 \end{bmatrix} \qquad \Sigma' = \begin{bmatrix} 25.19 & 0 \\ 0 & 237.89 \end{bmatrix}$$

Using equation (12) with P = .95, the rectangular parallelepiped region is given by:

$$x' = -21.19 \qquad x' = -43.64$$

$$y' = 1065.57 \qquad y' = 1134.57$$

Figure 7 reveals that with the proportion of the population in the tolerance region, P, specified to be .95, three samples out of 100 were rejected.

The value of P can be varied, but the results obtained in the rectangular parallelepiped tolerance region differ very slightly from the chi-square test. The complexity of transforming the mean and variance-covariance structure make the rectangular shape more difficult to implement. The chi-square test is the more powerful test and how it performs in other Monte Carlo studies will be the focus of later sections.

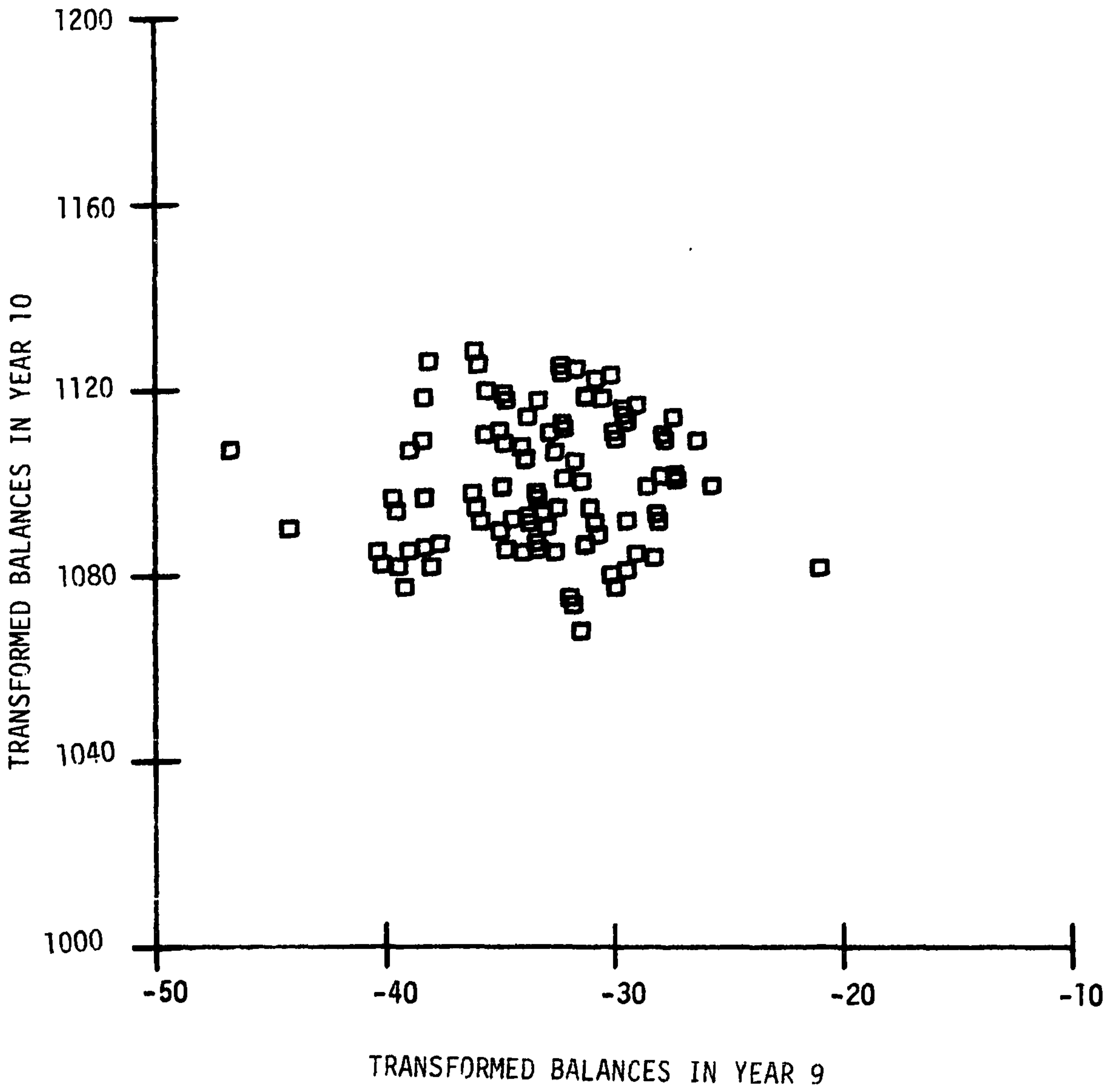

Fig. 6 Bivariate distribution of transformed balances in years 9 and 10 from an R1-10

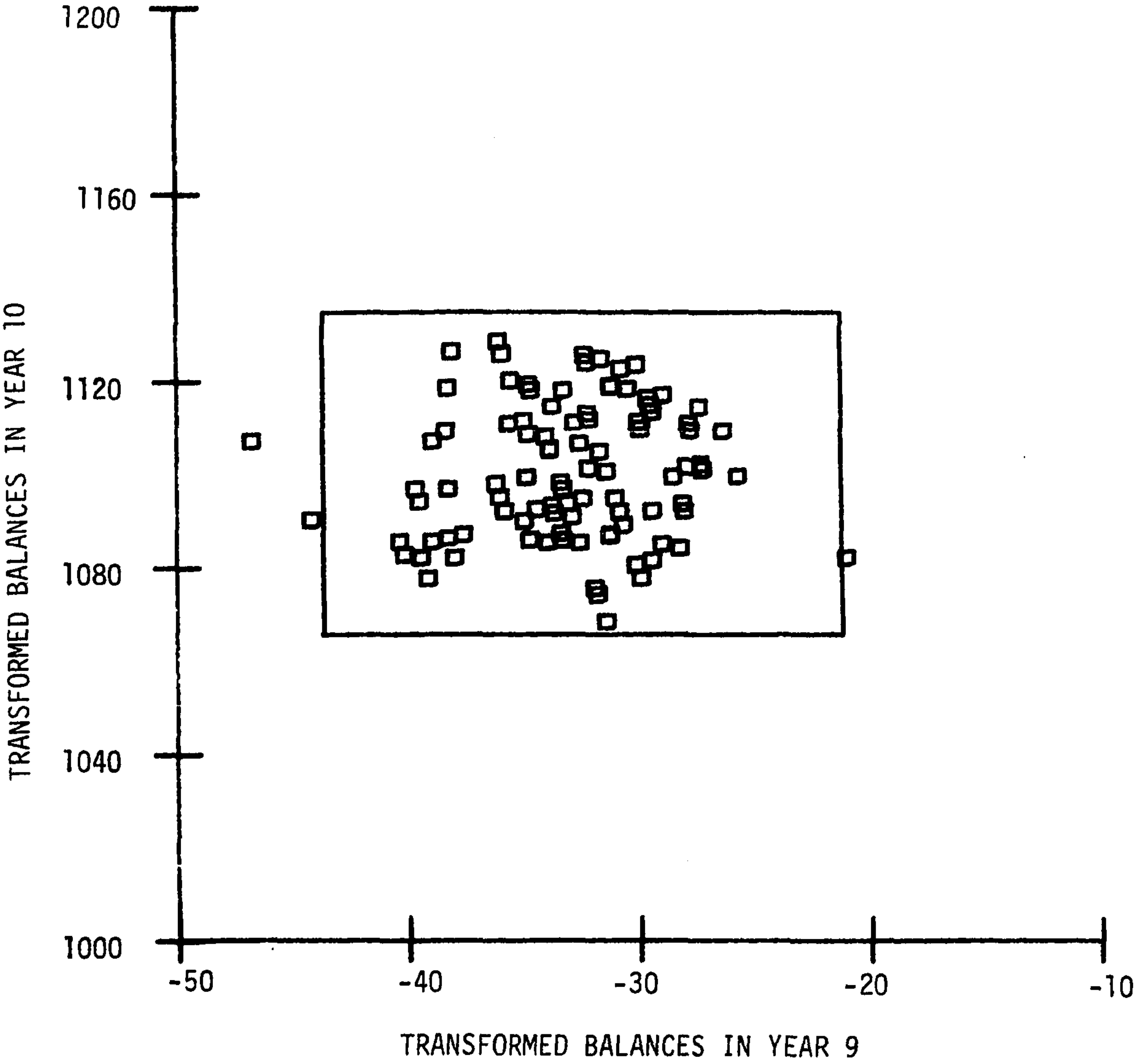

Fig. 7. Results of fitting rectangular parallelepiped region to transformed balances in years 9 and 10 from an R1-10

MONTE CARLO STUDIES IN A DATA-DEPENDENT SITUATION WITH NO GROWTH

One of the vital assumptions of the statistical model is that the occurrence of retirements from one vintage in no way affects the occurrence of retirements from another vintage. For the case of constant infusions, this assumption is fulfilled. However, when there are dependencies between the data during the period of study as is the case for most real world situations, the application of statistical theory assuming no data dependence causes rejection of the statistical hypothesis in most cases.

Application of Statistical Model Assuming No Data Dependence

To test the problems that occur when events are data-dependent, two Monte Carlo studies were made. In the first study, 50 random samples were taken from an L1 Iowa type curve with a ten year average service life. 1000 units priced at $1 each were placed at the beginning of the first year, and fifteen years of experience were generated. In the second Monte Carlo study, 100,000 units priced at $10 each were placed in service at the beginning of the first year. The dispersion, average service life, and period of study were the same as in the first study. Both studies specified a no growth situation. After retirements occur during the first year, the infusion is calculated so that the balance in subsequent years is as close as possible to the balance in service at the end of the first year. The decision rule used to determine the size of the infusion was discussed in the previous section. These random samples are documented in the Appendix, being samples 101 to 200 inclusive.

Examination of Table 3 reveals that in applying the statistical model the data are much more widely dispersed than theory would indicate for $1,000,000 initial placement. Five samples out of 50 were rejected by the chi-square test at the .05 level of significance using test years 11, 13, 15 for $1,000 initial installation. For $1,000,000 initial installation, 49 samples out of 50 were rejected by the chi-square test of hypothesis applied to test years 11, 13, and 15.

Table 3. Dispersion of the actual balances about the mean assuming no growth

Year	Number falling outside $\mu \pm 2\sigma$ for study with $1000 initial placement	Number falling outside $\mu \pm 2\sigma$ for study with $1,000,000 initial placement
11	0	32
13	0	33
15	0	39

For the $1,000,000 initial placement Monte Carlo study, rejection of the chi-square test in so many cases and the number of observations falling outside $\mu \pm 2\sigma$ lead one to doubt that the balances are normally distributed with means and variances calculated from Equations (6) and (9). This Monte Carlo study will be the focus in the remainder of this section. A theoretical approach to the problem of data dependence will be the aim of the next subsection.

Theoretical Model for Determining the Size of the Infusion and the Mean and Variance of the Balances

A variety of decision rules can be used to determine the size of the infusion to accomplish the goal of adding enough to have the target balance in service at the end of the year. The rule used by the PGM computer program uses events which will occur in determining the amount of the addition. The development set forth here conditions events on how many units were retired from the initial placement in the first year.

The following symbols will be used.

i = year of study where $i = 1, \ldots, m$.

m = maximum age when all units have been retired.

N = number of units in original installation.

M_i = number of units installed in year $i+1$.

X_{ij} = retirements that occur in year j from vintage i.

p_i = probability of retirement from time $i-1$ to i.

P = price per unit.

All the accounting entries will be made in dollar values.

Size of infusion

During the first year, N units are placed in service of which X_{11} are retired. The number of units to be added in the second year, M_1, is chosen such that the number of units left in service at the end of year 2 has expectation $N - X_{11}$ conditioned on X_{11}, or

$$E(\text{Number of units left at end of year two} | X_{11}) = N - X_{11} .$$

The number of units left in service at the end of year two is equal to

$$N - X_{11} + M_1 - X_{12} - X_{21} \quad .$$

M_1 is a function of some constant and multiplier times X_{11}, so substituting $A + BX_{11}$ for M_1, where A and B are unknown constants, yields

$$N - X_{11} + A + BX_{11} - X_{12} - X_{21} \quad .$$

We wish to solve

$$E(N - X_{11} + A + BX_{11} - X_{12} - X_{21} | X_{11}) = N - X_{11} \quad . \quad (13)$$

Using the facts that

$$E(X_{11} | X_{11}) = X_{11}$$

$$E(X_{12} | X_{11}) = (N - X_{11}) \frac{p_2}{1 - p_1}$$

$$E(X_{21} \; X_{11}) = (A + BX_{11})p_1$$

and substituting in Equation (13) yields the following solutions for A and B:

$$A = \frac{Np_2}{(1 - p_1)^2} \qquad B = \frac{-\,p_2}{(1 - p_1)^2} \quad .$$

Hence,

$$M_1 = (N - X_{11}) \frac{p_2}{(1 - p_1)^2} \quad . \tag{14}$$

For the third year, solution of M_2 is similar to the above procedure. The following equation must be solved

$$E(N - X_{11} - X_{12} - X_{13} + M_1 - X_{21} - X_{22} + M_2 - X_{31} | X_{11})$$
$$= N - X_{11} \quad . \tag{15}$$

Substituting in $M_2 = A + BX_{11}$ and use of the following results

$$E(X_{11}|X_{11}) = X_{11}$$

$$E(X_{1j}|X_{11}) = (N - X_{11}) \frac{p_j}{1 - p_1} \qquad j = 2, 3$$

$$E(X_{2j}|X_{11}) = (N - X_{11}) \frac{p_2 p_j}{(1 - p_1)^2} \qquad j = 1, 2$$

$$E(X_{31}|X_{11}) = (A + BX_{11})p_1 \tag{16}$$

produces a solution for (15):

$$M_2 = (N - X_{11})\left[\frac{p_3}{(1-p_1)^2} + \frac{p_2^2}{(1-p_1)^3}\right] \quad . \qquad (17)$$

In general to solve for the number of units left in service at the end of year k, one must solve

E(Number of units left in service at end of year $k | X_{11}$)

$$= E\left(N + \sum_{i=1}^{k-2} M_i + M_{k-1} - \sum_{i=1}^{k} \sum_{j=1}^{k-i+1} X_{ij} \middle| X_{11}\right) \quad . \qquad (18)$$

M_i for $i = 1, \ldots, k-2$ will be previously determined quantities and M_{k-1} will be solved for by substituting in $A + BX_{11}$ for M_{k-1} where A and B are unknown constants.

The conditional expectation can be found using the following results:

$$E(X_{11} | X_{11}) = X_{11}$$

$$E(X_{1j} | X_{11}) = (N - X_{11}) \frac{p_j}{(1 - p_1)} \qquad j = 2, \ldots, k$$

$$E(X_{ij} | X_{11}) = M_{i-1} p_j \qquad i = 2, \ldots, k-1; \quad j = 1, \ldots, k-i+1$$

$$E(X_{k1} | X_{11}) = (A + BX_{11}) p_1 \quad . \qquad (19)$$

Solutions for M_3, M_4, and M_5 are given below:

$$M_3 = (N - X_{11})\left[\frac{p_4}{(1-p_1)^2} + \frac{2p_2p_3}{(1-p_1)^3} + \frac{p_2^3}{(1-p_1)^4}\right] \qquad (20)$$

$$M_4 = (N - X_{11}) \left[\frac{p_5}{(1 - p_1)^2} + \frac{2p_2p_4 + p_3^2}{(1 - p_1)^3} + \frac{3p_2^2p_3}{(1 - p_1)^4} + \frac{p_2^4}{(1 - p_1)^5} \right] \tag{21}$$

$$M_5 = (N - X_{11}) \left[\frac{p_6}{(1 - p_1)^2} + \frac{2p_2p_5 + 2p_3p_4}{(1 - p_1)^3} + \frac{3p_2^2p_4 + 3p_2p_3^2}{(1 - p_1)^4} + \frac{4p_2^3p_3}{(1 - p_1)^5} + \frac{p_2^5}{(1 - p_1)^6} \right] \tag{22}$$

Equations (14), (17), (20), (21), and (22) give the decision rules to determine the number of units to be added in service at the beginning of years 2 through 6. The value in dollars of the additions would be the product of the price per unit, P, and the number of units placed in service in year i, M_{i-1}. Although it is possible to further specify the values of M_i, these results will be sufficient to examine the mean and variance structure of the balances with and without data dependence.

Mean and variance of balances

The mean and variance structure of the balances will be different from the nondata-dependent theory developed by White (1968). Since the additions have been chosen so that the expected number alive in any year conditioned on X_{11} is equal to $N - X_{11}$, use of the result from Hogg and Craig (1965)

$$E_y[E(X|Y)] = E(X) \tag{23}$$

where in this case $Y = X_{11}$ yields

$$E[\text{Balance in year } k] = PN(1 - p_1) \quad . \tag{24}$$

Let

$$A_1 = \frac{p_2}{(1 - p_1)^2}$$

$$A_2 = \frac{p_3}{(1 - p_1)^2} + \frac{p_2^2}{(1 - p_1)^3}$$

$$A_3 = \frac{p_4}{(1 - p_1)^2} + \frac{2p_2p_3}{(1 - p_1)^3} + \frac{p_2^3}{(1 - p_1)^4}$$

$$A_4 = \frac{p_5}{(1 - p_1)^2} + \frac{2p_2p_4 + p_3^2}{(1 - p_1)^3} + \frac{3p_2^2p_3}{(1 - p_1)^4} + \frac{p_2^4}{(1 - p_1)^5}$$

$$A_5 = \frac{p_6}{(1 - p_1)^2} + \frac{2p_2p_5 + 2p_3p_4}{(1 - p_1)^3} + \frac{3p_2^2p_4 + 3p_2p_3^2}{(1 - p_1)^4}$$
$$+ \frac{4p_2^3p_3}{(1 - p_1)^5} + \frac{p_2^5}{(1 - p_1)^6} \quad .$$

These M_i can be represented as

$$M_i = A_i(N - X_{11}) \qquad i = 1, \ldots, 5 \tag{25}$$

Variance for the occurrence of retirements from vintage i in year j, X_{ij}, can be calculated using the result given by Hogg and Craig (1965):

$$\mathrm{Var}\ Y = E[\mathrm{Var}(Y|X)] + \mathrm{Var}[E(Y|X)] \quad . \tag{26}$$

Hence,

$$\mathrm{Var}\ X_{11} = Np_1(1 - p_1)$$

$$\mathrm{Var}\ X_{1j} = E\left[(N - X_{11})p_j \frac{(1 - p_j)}{(1 - p_1)}\right] + \mathrm{Var}\left[\frac{(N - X_{11})p_j}{(1 - p_1)}\right]$$

$$= Np_j(1 - p_j) + \frac{Np_1p_j^2}{(1 - p_1)} \qquad j = 1, \ldots, m \tag{27}$$

$$\begin{aligned} \mathrm{Var}\ X_{ij} &= E[\mathrm{Var}(X_{ij}\ X_{11})] + \mathrm{Var}[E(X_{ij}\ X_{11})] \\ &= E[A_{i-1}(N - X_{11})p_j(1 - p_j)] \\ &\quad + \mathrm{Var}[(N - X_{11})A_{i-1}p_j] \\ &= A_{i-1}N(1 - p_j)p_j(1 - p_1) + A_{i-1}^2p_j^2Np_1(1 - p_1) \\ &\qquad\qquad i = 2, \ldots, 6 \\ &\qquad\qquad j = 1, \ldots, m \end{aligned} \tag{28}$$

Covariances for the above quantities can be related in a similar manner, since the occurrence of subsequent retirements depends on X_{11}. Hogg and Craig (1965) state:

$$\mathrm{Cov}(Y, Z) = E[\mathrm{Cov}(Y|X), \mathrm{cov}(Z|X)] + \mathrm{Cov}[E(Y|X), E(Z|X)] \ . \tag{29}$$

Since in this case the events depend upon X_{11}, (29) reduces to

$$\mathrm{Cov}(Y,Z) = \mathrm{Cov}[E(Y|X),\ E(Z|X)] \quad . \tag{30}$$

Hence,

$$\mathrm{Cov}(X_{11},\ X_{1j}) = -Np_1p_j \qquad j = 1, \ldots, m \tag{31}$$

$$\mathrm{Cov}(X_{11},\ X_{ij}) = -NA_{i-1}p_1(1 - p_1)p_j \qquad i = 2, \ldots, 6 \quad j = 1, \ldots, m \tag{32}$$

$$\mathrm{Cov}(X_{1j},\ X_{k\ell}) = NA_{k-1}p_1p_jp_\ell \qquad j = 1, \ldots m \quad k = 2, \ldots, 6 \quad \ell = 1, \ldots, m \tag{33}$$

$$\mathrm{Cov}(X_{ij},\ X_{k\ell}) = NA_{i-1}A_{k-1}p_1(1 - p_1)p_jp_\ell \tag{34}$$

$i, k = 2, \ldots, 6$

$j, \ell = 1, \ldots, m$

$i \neq k$ and $j \neq \ell$

Finding the variance for the balances in years 1 through 6 combines the above results. Let

$$B_m = 1 + \sum_{i=1}^{m-1} A_i \qquad m = 1, \ldots, 6 \tag{35}$$

$$\begin{aligned} \delta_{ij} &= B_m && \text{if} \quad i = j = 1 \\ &= 1 && \text{otherwise} \end{aligned} \tag{36}$$

$$\begin{aligned} k_{ij} &= i && \text{if} \quad i + j < n + 1 \\ &= i + 1 && \quad\ \ i + j = n + 1 \end{aligned} \tag{37}$$

$$\begin{aligned} \ell_{ij} &= j + 1 \qquad \text{if} \qquad i + j < n + 1 \\ &= i + 1 \qquad\qquad\quad i + j = n + 1 \end{aligned} \tag{38}$$

Thus,

$$\begin{aligned} \text{Var(Balance in year 1)} &= \text{Var}[P(N - X_{11})] \\ &= P^2 \text{ Var } X_{11} \\ &= P^2 N p_1 (1 - p_1) \end{aligned} \tag{39}$$

Var(Balance in year m)

$$\begin{aligned} &= \text{Var}\left[P(N + \sum_{i=1}^{m-1} M_i - \sum_{i=1}^{m} \sum_{j=1}^{m-i+1} X_{ij}) \right] \\ &= P^2 \left[\sum_{i=1}^{m} \sum_{j=1}^{m-i+1} \delta_{ij}^2 \text{ Var } X_{ij} \right. \\ &\quad \left. + 2 \sum_{i=1}^{m-1} \sum_{j=1}^{m-i+1} \sum_{k=k_{ij}}^{m} \sum_{\ell=\ell_{kj}}^{m} \delta_{ij} \text{ Cov}(X_{ij}, X_{k\ell}) \right] \end{aligned} \tag{40}$$

Comparison of Results Produced by Statistical Models With and Without Data Dependence

The decision rule for the size of the addition in each year which was derived in the previous subsection can be compared to what was actually produced by the Monte Carlo study. The additions produced by the computer, on the average, slightly understate the expected addition computed from formulas (14), (17), (20), (21), and (22).

Table 4. Comparison of additions produced by Monte Carlo study with additions expected under the statistical model

Year	Mean of additions from Monte Carlo study	Addition expected when events are conditioned	% Diff.
2	14,526	14,530	.006
3	28,033	28,130	.32
4	44,597	44,750	.34
5	61,311	61,370	.09
6	74,628	75,890	1.6

The means and variances of the balances produced by the two statistical models are compared in Table 5. The average of the mean balance assuming no data dependence and sample mean of the variance of balances assuming no data dependence were found by averaging the values produced by Equations (6) and (9) respectively to each Monte Carlo sample. The means and variances for the data-dependent case are taken from applying Equations (24), (27), and (28).

The Monte Carlo study produces means (simulated balances) assuming no data dependence which in the years examined understate the target balance from Equation (24). The variances of the balances obtained assuming no data dependence neglect the fact that the price per unit, P, is a constant multiplied times the number of units involved at a given point in time.

Table 5. Comparison of means and variances produced by the two statistical models

	No data dependence		Data dependence			
Year	Average of mean of balances	Average of variance of balances	Mean balance expected	Variance of balance expected	% Diff. Means	% Diff. Var.
1	996,385	10,843	996,370	106,567	0	882
2	996,375	17,824	996,370	179,305	0	906
3	996,292	44,155	996,370	445,320	0	909
4	996,148	82,927	996,370	831,370	0	903
5	996,091	130,056	996,370	1,360,598	0	946
6	996,098	178,442	996,370	2,086,224	0	1069

The variance formula (9) given in White's calculations assumes the additions N_j may be in units or dollars. If the additions and balances are in dollar values, the dollar values are the products of the number of units and the price per unit. This qualification results in the following modification to White's work:

$$E(B_k) = P \sum_{i=1}^{k} N_i \left(1 - \sum_{j=1}^{k-i+1} \pi_j\right) \tag{41}$$

$$\mathrm{Var}(B_k) = P^2 \sum_{i=1}^{k} N_{k-i+1} \left(1 - \sum_{j=1}^{i} \pi_j\right)\left(\sum_{j=1}^{i} \pi_j\right) \tag{42}$$

$$\mathrm{Cov}(B_k\ B_{k'}) = P^2 \sum_{i=1}^{k} N_{k-i+1} \left(\sum_{j=1}^{i} \pi_j\right)\left(1 - \sum_{j=1}^{k'-k+i} \pi_j\right) \tag{43}$$

The expectation does not differ from what White found in Equation (6). However, the variances and covariances in (42) and (43) are P times the results in Equations (9) and (10) respectively.

Taking White's nondata-dependent theory and multiplying the variance given in Equation (9) by P would produce substantial changes in the dispersion of the balances with respect to the mean and variance. Table 3 revealed in applying Equation (9), the observations are more dispersed than theory would expect. If the variances in (9) were recalculated using (42), 6, 6, and 4 balances for test years 11, 13, and 15 respectively would fall outside $\mu \pm 2\sigma$. This step is a dramatic improvement over the results in Table 3. A chi-square test applied using the revised variances and covariances would result in rejection of 6 out of 50 samples at the .05 level, a great improvement over the rejection of 49 out of 50 samples previously.

In the real world, the determination of the price per unit could make implementation difficult. This problem coupled with the fact that modeling with data dependence produces additional covariance terms which go to zero if there is no data dependence causing this modeling to produce higher variances than in the other case.

When applying nondata-dependent theory developed by White (1968), the additions were assumed to be constants with no variability. As can be seen from Table 6, the actual additions in this case have a wide variability, even greater than the variability of the balances.

Table 6. Sample means and sample standard deviations for actual additions and actual balances

	Additions		Balances	
Year	Sample mean	Sample standard deviation	Sample mean	Sample standard deviation
1	1,000,000	0	996,329	166.9
2	14,526	311.3	996,334	169.8
3	28,033	540.4	996,329	174.9
4	44,597	659.1	996,319	187.7
5	61,311	855.4	996,312	207.4
6	74,628	718.9	996,309	203.6
7	81,993	756.1	996,302	208.7
8	86,126	868.2	996,287	222.5
9	89,429	812.3	996,281	216.3
10	92,984	931.0	996,281	225.6
11	95,606	850.6	996,274	241.0
12	97,831	920.8	996,258	262.6
13	99,530	967.5	996,271	259.6
14	100,847	975.4	996,268	263.5
15	101,534	1071.9	996,242	267.2

The nondata-dependent theory produces a mean and variance for the balance each year for each sample. Comparison with data-dependent theory in Table 5 has shown that for this Monte Carlo study the variances of the balances assuming no data dependence need to be larger. To determine what the variances should be, the variability of the estimates of the mean and variances calculated from Equations (6) and (9) is compared in Table 7 for selected years.

Table 7. Sample means and sample standard deviations for means and variances of balances assuming no data dependence

	Mean of balances assuming no data dependence		Variance of balances assuming no dependence	
Year	Mean	Standard deviation	Mean	Standard deviation
5	996,091	1798.3	130,077	41.27
7	996,199	1909.22	221,242	121.8
11	997,529	1921.1	310,246	354.9
13	997,564	1801.9	322,175	427.6
15	997,645	1972.2	316,090	457.4

The sample variance of the mean assuming no data dependence should be a more accurate reflection of what the variance in this situation should be than the variances obtained from Equation (9). To relate these quantities, a regression analysis was performed for

$$Y = \alpha + \beta X + \varepsilon \tag{44}$$

where

Y = sample variance of the mean obtained from Equation (6).

X = sample mean of the variance obtained from Equation (9).

ε = error term.

The estimates using the data in Table 7 are $\hat{\alpha}$ = 2.98977 E 06 and $\hat{\beta}$ = 2.0348.

Table 8 reveals that the F test is not significant. Even though this regression attempt to relate the two quantities has not been successful the same approach can be taken with the covariance terms.

Table 8. Results of regression analysis for variance

Source	df	SS	MS	F
Regression	1	1.16 E 11	1.16 E 11	1.27
Residual	3	2.73 E 11	9.09 E 10	
Total		3.89 E 11		

Table 9 gives comparisons for the covariance terms in years 11, 13, and 15. Covariances between years i and j can be estimated by the Monte Carlo study via the following relationship:

$$\text{cov}(B_k \ B_{k'}) = \Sigma X_i' Y_i' - n \bar{X}' \bar{Y}' \qquad (45)$$

where

B_k, $B_k{}'$ = balance in years k and k' respectively.

X_i' = estimate of mean in year k for sample i produced from Equation (6).

Y_i' = estimate of mean in year k' for sample i produced from Equation (6).

n = number of samples.

$$\bar{X}' = \sum_{i=1}^{n} X_i' .$$

$$\bar{Y}' = \sum_{i=1}^{n} Y_i' .$$

Table 9. Comparison of covariance terms for balance

Years	Covariance calculated from (45)	Sample mean of covariances from (10)
11, 13	3.3554 E 08	230,949
11, 15	3.6909 E 08	160,337
13, 15	3.0198 E 08	228,752

The quantities in Table 9 can be related by a regression equation

$$Y = \alpha + \beta X + \varepsilon \tag{46}$$

where

Y = covariance terms calculated from (45).

X = mean of covariance terms calculated from (10).

ε = error term.

This produces $\hat{\alpha} = -8114.96$ and $\hat{\beta} = 91.35$.

This analysis given in Table 10 is no more successful in producing a significant F test. While the number of observations in each case is limited, this procedure is a workable method of determining what variances and covariances should be in a data-dependent situation using nondata-dependent formulas.

Table 10. Regression analysis for covariance terms

Source	df	SS	MS	F
Regression	1	2.6804 E 07	2.6804 E 07	2.48
Residual	1	1.8007 E 07	1.8007 E 07	
Total	1	3.7611 E 07		

Problems in Implementing Computer Decision Rule

The decision rule used by the PGM computer program uses events which will logically occur in the future to determine what the infusion in each year will be. Since this is the case, a statistical modeling of the process the computer goes through in data-dependent situations cannot be achieved. However, the additions and balances the computer produces can be compared to what one would expect.

For example, in year 2 the addition in dollar values is $P\ X_{12}/(1 - p_1)$.

$$E(\text{Additions in year 2}) = E\left[P\left(\frac{X_{12}}{1 - p_1}\right)\right] = \frac{P}{1 - p_1} E(X_{12})$$

$$= \frac{N\ P\ p_2}{1 - p_1} \tag{47}$$

$$E(\text{Balance in year 2}) = E[P(N - X_{11} - X_{12} + M_1 - X_{21})]$$

$$= P[N(1 - p_1 - p_2) + \frac{N\ p_2}{1 - p_1}(1 - p_1)]$$

$$= PN(1 - p_1) \tag{48}$$

This process can be repeated for each year. The results of computing these expectations for each year are found in Table 11. The average of the additions produced by the computer given in Table 6 are remarkably close to what one would expect them to be. However, the computer produces a smaller balance on the average as shown in Table 6 than one would expect. In the course of computing retirements for each vintage, it appears the retirements X_{ij} are being overstated by the computer. It is recommended that further scrutiny be given to the PGM program to examine the process used to simulate retirements.

Table 11. Expected additions and balances using the computer decision rule

Year	Expected addition	Expected balance
2	14,570	996,270
3	28,120	996,420
4	44,740	996,420
5	61,370	996,420
6	74,630	996,410
7	82,040	996,410
8	85,920	996,420
9	89,520	996,420
10	92,810	996,420
11	95,670	996,430
12	98,000	997,830
13	99,710	997,920
14	100,980	997,560
15	101,880	997,480

ABILITY OF THE BALANCES METHOD TO DETECT CORRECT DISPERSION AND AVERAGE SERVICE LIFE

Many depreciation analysts use the simulated-plant record balances method to select the correct dispersion and average service life. The choice is made by picking the retirement dispersion and average service life which minimize the sum of squares differences between the actual and simulated balances. Some practitioners feel that the combination of dispersion and average service life which produces this minimum sum of squares is a unique best-fitting representation of mortality characteristics.

To examine this notion, the Monte Carlo studies discussed in the previous sections were analyzed using the SPR balances method and the chi-square statistic. A computer program was written to analyze these data combining work done by White and Cowles (1972) and White (1968). The program documented by White and Cowles (1972) performs the SPR analysis, giving the average service life which minimizes the sum of squares difference between actual and simulated balances for each Iowa type curve. The program developed by White (1968) finds the chi-square statistic after the dispersion and average service life are input. The resulting computer program determines the chi-square test statistic for each of the 31 Iowa type curves using the test years specified by the balances method.

Since the data in these Monte Carlo studies are very regular with little variation built in, they cannot be supposed to reflect a real world situation which is affected by factors such as inflation, price variation, or economic uncertainty. Nevertheless, from analyzing the Monte Carlo studies, the idea of producing a unique best-fitting representation of the data can be clarified or dispelled.

Monte Carlo Study With Constant Infusions

These data were generated from an R1 Iowa type curve with a ten year average service life. One hundred units priced at $1 each were placed in service every year for ten years. These samples are listed in the Appendix, consisting of samples 1 to 100. Two samples out of 100 were rejected by the chi-square statistic at the .05 level using the R1 type curve and average service life produced by the sum of squares criterion.

In spite of the regularity of these data, the balances method seldom selected the R1-10 curve from which the data are known to be generated. In Table 12, a distribution of service lives for the R1 curve produced by the sum of squares criterion is given.

This vairation from the true average service life by as much as ±10% is surprising and could have a significant effect on the depreciation accrual for a property with a longer service life.

Table 12. Number of samples generating the given average service life for an R1 curve by the SPR balances method for samples 1 through 100

ASL	Frequency
9.2	1
9.3	1
9.4	4
9.5	7
9.6	8
9.7	8
9.8	13
9.9	7
10.0	5
10.1	4
10.2	9
10.3	7
10.4	5
10.5	6
10.6	7
10.7	4
10.8	0
10.9	4

In Table 13, it is found that the R1 dispersion is not always the curve which produces a minimum sum of squares. In fact in only 10% of the samples did the R1 curve produce the minimum sum of squares differences. The sum of squares criterion has the R1 curve ranked as low as eleventh out of the 31 Iowa type curves. The results obtained from the chi-square statistic do not produce ranking that differs greatly from the sum of squares criterion.

Monte Carlo Studies With Data Dependence

Samples 101 through 150 in the Appendix were generated from an L1-10 Iowa type corve. One thousand units priced at $1 each were placed in service in year one, and each sample was specified to have no growth. Using test years 11, 13, and 15, the chi-square statistic assuming no data dependence rejected 5 out 50 samples when examining the L1 curve and average service life produced by the sum of squares criterion.

Table 14 gives the distribution of service lives produced for the L1 curve by the sum of squares criterion. In this case, the average service life varies up to ±5% from the known average service life. Again if this property had a longer average service life, such a discrepancy in estimating the average service life could play havoc with the depreciation rate.

Table 13. Chi-square ranking compared to sum of squares ranking for R1 curve for samples 1 through 100

Sum of squares ranked in ascending order for R1 curve for samples 1 through 100	Chi-square statistic ranked in ascending order for R1 curve for samples 1 through 100												
	1	2	3	4	5	6	7	8	9	10	11	12	
1	10												10
2	1	9	1										11
3		2	6										8
4				6	1								7
5					5	1							6
6						14	2						16
7							6	1					7
8							2	22					24
9									7				7
10										2			2
11											1	1	2
12													0
	11	11	7	6	6	15	10	23	7	2	1	1	100

Table 14. Number of samples generating the given average service life for an L1 curve by the SPR balances method for samples 101 through 150

ASL	Frequency
9.6	1
9.7	5
9.8	3
9.9	8
10.0	11
10.1	10
10.2	7
10.3	2
10.4	3

Table 15 compares results produced by the sum of squares criterion with those produced by the chi-square statistic. In this case only 5 out of 50 samples produced the minimum sum of squares differences for the L1 dispersion. The L1 dispersion actually ranks as low as tenth by both sum of squares and chi-square.

For the other no growth Monte Carlo study with 100,000 units priced at $10 each, the results were different. The chi-square statistic rejected 49 out of 50 samples at the .05 level. However, for all fifty samples the chi-square statistic and sum of squares differences were a minimum for the L1 dispersion. The average service life for the L1 dispersion produced by the sum of squares criterion was 10.0 in every case.

Table 15. Chi-square ranking compared to sum of squares ranking for L1 dispersion for samples 101 through 150

Sum of squares ranked from smallest to largest for L1 curve	Chi-square ranked in ascending order for L1 curve										
	1	2	3	4	5	6	7	8	9	10	
1	4	1									5
2		7	3								10
3		1	3	3							7
4				5	1						6
5			1	1	10		1				13
6						1					1
7						1	2				3
8							1	2			3
9									1		1
10										1	1
	4	9	7	9	11	2	4	2	1	1	50

Discussion

While the first no growth Monte Carlo study produced more statistically acceptable results, the SPR balances method had difficulty in selecting the correct choice of dispersion and average service life. In many cases, a large number of curves were grouped near the curve which produced a minimum sum of squares difference with little difference in their respective sum of squares differences and even less in the indices of variation. More accurate conclusions would be gained by subjectively analyzing several curves as to the average service life and position and height of the mode of the density function.

When given Monte Carlo studies where the values were smaller in a hundred or thousand dollar units, the balances method had greater difficulty in selecting the proper dispersion than when the additions were larger. The chi-square test however performed better in the former case than in the latter. The problems associated with the chi-square modeling were discussed at length in the section involving data dependence. If the chi-square modeling incorporated the price per unit P as a separate quantity from the additions the analysis could be greatly improved. Whether this phenomena would occur in other cases by varying the size of the initial installation would be a subject of great interest for further research.

SUMMARY AND CONCLUSIONS

The simulated plant-record method is the only technique used to analyze semi-actuarial data which gives estimates of both the retirement dispersion and average service life. Practitioners in industry have relied upon the SPR method to produce a unique best fitting curve. In the past statistical modeling has achieved limited success in analyzing the SPR method. Exploration of these problems has produced the following conclusions:

(i) When the data are not dependent on the occurrence of retirements from one vintage to the next, the statistical modeling proposed by White is appropriate. The balances are normally distributed with means and variances given in equations (6) and (9) respectively.

(ii) When varying the shape of the tolerance region from ellipsoidal to a rectangular parallelepiped, little change is found. For ease of computation, the ellipsoidal shape is preferred.

(iii) When examining data-dependent situations, White's modeling produces more frequent rejection of the Monte Carlo samples than theory would expect.

(iv) If one were to separate the price per unit from the dollar value of each account, the data-dependent case would produce larger variances and covariances and could be analyzed more successfuly by White's model.

(v) In the data-dependent case variability of the means produced by the Monte Carlo studies can be used to estimate true variability for data-dependent observations.

(vi) The SPR balances method does not produce a unique best fitting curve. Instead several curves should be analyzed before choosing one as a best fit.

(vii) On the basis of these data discussed herein, it appears the SPR balances method estimates the correct dispersion and average service life with greater precision when the dollar values involved are of greater magnitude than when the dollar values are smaller in magnitude.

The conclusions drawn from this study raise a number of questions that warrant further investigation. The assumption that all vintages come from a homogeneous population has not been explored in this study. However, for real world application, this assumption is almost certainly violated. Modifications to the statistical development presented would be of interest.

Separating the price component from the analysis for computation of the variance and covariance could present quite a problem in industrial applications. Choice of a variable price for each year could be used to approximate conditions such as inflation or economic uncertainty.

The PGM computer program was found to produce more retirements than could be expected theoretically. The decision rule in data-dependent cases for each year's addition closely approximates the additions produced by the Monte Carlo studies. However, before any further studies are made using this program, a better understanding of the logic should be undertaken.

The statistical modeling presented for data-dependent observations is conditioned on retirements which occur during the first year. Conditioning on retirements which occur in later years would be a logical extension of this work. This process would be of limited value for real world situations.

Further, Monte Carlo studies in other situations than the no growth would be of aid in determining how the balances method is affected by the size of initial installation and price per unit. Other areas which could be explored are quantifying a scale for the index of variation or empirically substantiating the scales given for the conformance index or retirements experience index.

REFERENCES

Alexander, H. W. 1961. Elements of mathematical statistics. John Wiley and Sons, Inc., New York.

Anderson, R. L., and T. A. Bancroft. 1952. Statistical theory in research. McGraw Hill Book Co., Inc., New York.

Bauhan, A. E. 1947. Life analysis of utility plant for depreciation accounting purposes by the simulated plant-record method. Unpublished multilithed paper presented at National Conference of Electric and Gas Utility Accountants, American Gas Association, Edison Electric Institute, Buffalo, New York.

Bauhan, A. E. 1948. Simulated plant-record method of life analysis of utility plant for depreciation accounting purposes. Land Economics 24: 129-136.

Brennen, J. F. 1959. Mortality curves for physical plant. National Conference of Electric and Gas Utility Accountants Proceedings 1959: 323-356.

Brennen, J. F. 1957. Plant mortality. Electrical Engineering 76, No. 2: 131-134.

Caunt, W. H. 1960. Depreciation calculations with a high-speed computer. Unpublished multilithed paper presented at National Conference of Electric and Gas Utility Accountants, New York.

Chew, V. 1966. Confidence, prediction, and tolerance regions for the multivariate normal distribution. Journal of the American Statistical Association 66: 605-617.

Couch, F. 1957. Classification of type O retirement characteristics of industrial property. Unpublished M.S. thesis. Library, Iowa State University of Science and Technology, Ames, Iowa.

Cowles, H. A. 1957. Prediction of mortality characteristics of industrial property groups. Unpublished Ph.D. Thesis. Library, Iowa State University of Science and Technology, Ames, Iowa.

Edison Electric Institute. 1952. Methods of estimating utility plant life. Edison Electric Institute, New York.

Erbe, H. C. 1971. A comparison of actuarial and simulated plant-record methods of property account analysis. Unpublished M.S. Thesis. Library, Iowa State University of Science and Technology, Ames, Iowa.

Fitch, W. C., F. K. Wolf, and B. H. Bissinger. 1975. The estimation of depreciation. Center for Depreciation Studies, Western Michigan University, Kalamazoo, Michigan.

Fraser, D. A. S., and I. Guttman. 1956. Tolerance regions. Annals of Mathematical Statistics 27: 162-179.

Freund, J. E. 1967. Modern elementary statistics. 3rd ed. Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Garland, W. D. 1967. Proposal of a simulated plant record period retirements method for plant mortality analysis. Unpublished multilithed paper presented at National Conference of Electric and Gas Utility Accountants, Los Angeles, California.

Hill, C. G. 1922a. Depreciation of telephone plants. 1. Telephone 82, No. 11: 12-16.

Hill, C. G. 1922b. Depreciation of telephone plants. 2. Telephony 82, No. 12: 21-26.

Hogg, R. V., and A. T. Craig. 1965. Introduction to mathematical statistics. 2nd ed. The Macmillan Company, New York.

International Mathematical and Statistical Libraries. 1977. The IMSL Library. IMSL, Houston, Texas.

John, S. 1963. Tolerance regions for multivariate normal distributions. Sankhya 25: 363-368.

Krane, S. A. 1963. Analysis of survival data by regression techniques. Technometrics 5, No. 9: 161-174.

Lamp, G. E. 1968. Dispersion effects in industrial property life analysis. Unpublished Ph.D. Thesis. Library, Iowa State University of Science and Technology, Ames, Iowa.

Marston, A., R. Winfrey, and J. C. Hempstead. 1970. Engineering valuation and depreciation. 5th ed. The Iowa State University Press, Ames, Iowa.

National Association of Railroad and Utility Commissioners. 1943. Report of committee on depreciation. National Association of Railroad and Utility Commissioners, Washington, D. C.

National Association of Regulatory Utility Commissioners. 1968. Public utility depreciation practices. National Association of Regulatory Utility Commissioners, Washington, D. C.

Rippe, R. D. 1969. Service life expectation of institutional china tableware by life analysis. Unpublished M.S. Thesis. Library, Iowa State University of Science and Technology, Ames, Iowa.

Rose, R. G. 1972. Life analysis of institutional china tableware. Unpublished M.S. Thesis. Library, Iowa State University of Science and Technology, Ames, Iowa.

Russo, J. G. 1978. Iowa type survivor curve revalidation. Unpublished Ph.D. Thesis. Library, Iowa State University of Science and Technology, Ames, Iowa.

Schneider, H., and G. P. Barker. 1973. Matrices and linear algebra. 2nd ed. Holt, Rinehart, and Winston, Inc., New York.

Siotani, M. 1964. Tolerance regions for a multivariate normal population. Annals of the Institute of Statistical Mathematics 16: 135-153.

Snedecor, G. W., and W. G. Cochran. 1967. Statiscal methods. 6th ed. The Iowa State University Press, Ames, Iowa.

Strecok, A. J. 1968. On the calculation of the inverse of the error function. Mathematics of Computations 22: 144-158.

Thoni, H. 1967. Transformations of variables used in the analysis of experimental and observation data. A review. Technical Report No. 7, Statistical Laboratory, Iowa State University, Ames, Iowa.

White, R. E. 1968. The miltivariate normal distribution and the simulated plant-record method of life analysis. Unpublished M.S. Thesis. Library, Iowa State University of Science and Technology, Ames, Iowa.

White, R. E., and H. A. Cowles. 1970. A test procedure for the simulated plant-record method of life analysis. Journal of the American Statistical Association 70: 1204-1212.

White, R. E., and H. A. Cowles. 1972. Simulated plant-record survivor analysis program. Special Report. Iowa State University Engineering Research Institute, Ames, Iowa.

Whiton, H. R. 1947. The indicated retirement approach to the simulated plant-record method of estimating lives of mass accounts of utility property for depreciation accounting purposes. Unpublished multilithed paper presented at National Conference of Electric and Gas Utility Accountants, American Gas Association, Edison Electric Institute, Buffalo, New York.

Winfrey, R. 1942. Depreciation of group properties. Iowa State College Engineering Experiment Station, Bulletin 155.

Winfrey, R. 1967. Statistical analyses of industrial property retirements. Iowa State University Engineering Research Institute Bulletin 125. Revised edition.

ACKNOWLEDGMENTS

The author is deeply indebted to Dr. Harold A. Cowles who fostered my interest in the field of engineering valuation and who provided advice, counsel, encouragement, and consolation during the course of this research. The statistical development presented herein owes much to the insight and interest of Dr. Herbert T. David.

I am grateful to Dr. Keith L. McRoberts and Dr. Chester W. Fitch of Western Michigan University who generously provided computer funds without which this research could not have been completed. Thanks go to Dr. Aly A. Mahmoud and the Power Affiliate Program which furnished financial support of this investigation.

I am indebted to my parents, Charles G. and Florence Sledge Hallaman, who have provided encouragement and support throughout my academic career.

Finally, my deepest thanks and appreciation go to my husband, Wendell, who in the course of pursuing his own research offered patience, understanding, and encouragement during all phases of this study.

APPENDIX: LISTING OF SAMPLES GENERATED FOR MONTE CARLO STUDIES

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
1	1	100	2	98
	2	100	3	195
	3	100	5	290
	4	100	10	380
	5	100	26	454
	6	100	24	530
	7	100	23	607
	8	100	23	684
	9	100	41	743
	10	100	42	801
2	1	100	1	99
	2	100	3	196
	3	100	4	292
	4	100	11	381
	5	100	15	466
	6	100	13	533
	7	100	27	626
	8	100	25	701
	9	100	42	759
	10	100	47	812
3	1	100	1	99
	2	100	6	193
	3	100	1	292
	4	100	13	379
	5	100	15	464
	6	100	16	548
	7	100	31	617
	8	100	32	685
	9	100	32	753
	10	100	36	817
4	1	100	0	100
	2	100	2	198
	3	100	4	294
	4	100	7	387
	5	100	13	474
	6	100	15	559
	7	100	30	629
	8	100	26	703
	9	100	32	771
	10	100	49	822

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
5	1	100	1	99
	2	100	3	196
	3	100	5	291
	4	100	3	388
	5	100	10	478
	6	100	33	545
	7	100	32	613
	8	100	31	682
	9	100	29	753
	10	100	54	799
6	1	100	1	99
	2	100	4	195
	3	100	7	288
	4	100	12	376
	5	100	10	466
	6	100	15	551
	7	100	29	622
	8	100	26	696
	9	100	36	760
	10	100	51	809
7	1	100	2	98
	2	100	4	194
	3	100	5	289
	4	100	7	382
	5	100	19	463
	6	100	16	547
	7	100	23	624
	8	100	20	704
	9	100	38	766
	10	100	50	816
8	1	100	2	98
	2	100	6	192
	3	100	7	285
	4	100	9	376
	5	100	12	464
	6	100	20	544
	7	100	28	616
	8	100	29	687
	9	100	42	745
	10	100	54	791

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
9	1	100	3	97
	2	100	3	194
	3	100	7	287
	4	100	7	380
	5	100	13	467
	6	100	20	547
	7	100	25	622
	8	100	27	695
	9	100	43	752
	10	100	42	810
10	1	100	1	99
	2	100	4	195
	3	100	10	285
	4	100	6	379
	5	100	12	467
	6	100	26	541
	7	100	24	617
	8	100	20	697
	9	100	28	769
	10	100	46	823
11	1	100	3	97
	2	100	7	190
	3	100	10	280
	4	100	9	371
	5	100	11	460
	6	100	15	545
	7	100	24	621
	8	100	37	684
	9	100	56	728
	10	100	45	783
12	1	100	2	98
	2	100	2	196
	3	100	11	285
	4	100	13	372
	5	100	12	460
	6	100	18	542
	7	100	27	615
	8	100	28	687
	9	100	41	746
	10	100	47	799

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
13	1	100	3	97
	2	100	3	194
	3	100	3	291
	4	100	11	380
	5	100	12	468
	6	100	19	549
	7	100	28	621
	8	100	35	686
	9	100	40	746
	10	100	36	810
14	1	100	0	100
	2	100	2	198
	3	100	6	292
	4	100	12	380
	5	100	10	470
	6	100	21	549
	7	100	23	626
	8	100	33	693
	9	100	34	759
	10	100	46	813
15	1	100	0	100
	2	100	4	196
	3	100	9	287
	4	100	6	381
	5	100	15	466
	6	100	20	546
	7	100	28	618
	8	100	39	679
	9	100	34	745
	10	100	46	799
16	1	100	0	100
	2	100	4	196
	3	100	5	291
	4	100	7	384
	5	100	9	475
	6	100	21	554
	7	100	28	626
	8	100	30	696
	9	100	42	754
	10	100	45	809

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
17	1	100	2	98
	2	100	5	193
	3	100	4	289
	4	100	8	381
	5	100	13	468
	6	100	16	552
	7	100	25	627
	8	100	28	699
	9	100	34	765
	10	100	46	819
18	1	100	2	98
	2	100	6	192
	3	100	6	286
	4	100	15	371
	5	100	17	454
	6	100	15	539
	7	100	30	609
	8	100	31	678
	9	100	34	744
	10	100	45	799
19	1	100	2	98
	2	100	3	195
	3	100	7	288
	4	100	20	368
	5	100	20	448
	6	100	19	529
	7	100	21	608
	8	100	32	676
	9	100	27	749
	10	100	45	804
20	1	100	0	100
	2	100	2	198
	3	100	4	294
	4	100	16	378
	5	100	19	459
	6	100	23	536
	7	100	17	619
	8	100	34	685
	9	100	36	749
	10	100	49	800

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
21	1	100	0	100
	2	100	6	194
	3	100	6	288
	4	100	12	376
	5	100	14	462
	6	100	26	536
	7	100	21	615
	8	100	27	688
	9	100	41	747
	10	100	49	798
22	1	100	0	100
	2	100	6	194
	3	100	7	287
	4	100	10	377
	5	100	17	460
	6	100	16	544
	7	100	26	618
	8	100	26	692
	9	100	34	758
	10	100	38	820
23	1	100	1	99
	2	100	5	194
	3	100	14	280
	4	100	8	372
	5	100	20	452
	6	100	21	531
	7	100	26	605
	8	100	22	683
	9	100	43	740
	10	100	43	797
24	1	100	2	98
	2	100	1	197
	3	100	8	289
	4	100	9	380
	5	100	18	462
	6	100	20	542
	7	100	31	611
	8	100	19	692
	9	100	41	751
	10	100	49	802

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
25	1	100	1	99
	2	100	3	196
	3	100	8	288
	4	100	11	377
	5	100	16	461
	6	100	23	538
	7	100	30	608
	8	100	28	680
	9	100	39	741
	10	100	38	803
26	1	100	1	99
	2	100	2	197
	3	100	4	293
	4	100	18	375
	5	100	14	461
	6	100	25	536
	7	100	17	619
	8	100	27	692
	9	100	38	754
	10	100	39	815
27	1	100	4	96
	2	100	4	192
	3	100	6	286
	4	100	9	377
	5	100	22	455
	6	100	17	538
	7	100	31	607
	8	100	31	676
	9	100	40	736
	10	100	39	797
28	1	100	0	100
	2	100	5	195
	3	100	4	291
	4	100	14	377
	5	100	15	462
	6	100	24	538
	7	100	23	615
	8	100	28	687
	9	100	30	757
	10	100	43	814

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
29	1	100	1	99
	2	100	4	195
	3	100	4	291
	4	100	10	381
	5	100	8	473
	6	100	24	549
	7	100	27	622
	8	100	31	691
	9	100	26	765
	10	100	63	802
30	1	100	1	99
	2	100	3	196
	3	100	8	288
	4	100	18	370
	5	100	16	454
	6	100	21	533
	7	100	29	604
	8	100	41	663
	9	100	31	732
	10	100	45	787
31	1	100	1	99
	2	100	6	193
	3	100	7	286
	4	100	14	372
	5	100	13	459
	6	100	17	542
	7	100	23	619
	8	100	27	692
	9	100	46	746
	10	100	46	800
32	1	100	0	100
	2	100	1	199
	3	100	5	294
	4	100	8	386
	5	100	20	466
	6	100	14	552
	7	100	21	631
	8	100	29	702
	9	100	37	765
	10	100	41	824

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
33	1	100	1	99
	2	100	3	196
	3	100	2	294
	4	100	18	376
	5	100	17	459
	6	100	17	542
	7	100	22	620
	8	100	21	699
	9	100	37	762
	10	100	43	819
34	1	100	0	100
	2	100	6	194
	3	100	1	293
	4	100	14	379
	5	100	24	455
	6	100	24	531
	7	100	27	604
	8	100	29	675
	9	100	24	751
	10	100	47	804
35	1	100	1	99
	2	100	4	195
	3	100	11	284
	4	100	6	378
	5	100	15	463
	6	100	26	537
	7	100	24	613
	8	100	23	690
	9	100	44	746
	10	100	56	790
36	1	100	3	97
	2	100	2	195
	3	100	6	289
	4	100	12	377
	5	100	17	460
	6	100	25	535
	7	100	28	607
	8	100	38	669
	9	100	27	742
	10	100	40	802

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
37	1	100	2	98
	2	100	8	190
	3	100	9	281
	4	100	6	375
	5	100	11	464
	6	100	21	543
	7	100	20	623
	8	100	33	690
	9	100	27	763
	10	100	39	824
38	1	100	1	99
	2	100	5	194
	3	100	9	285
	4	100	13	372
	5	100	13	459
	6	100	12	547
	7	100	28	619
	8	100	36	683
	9	100	39	744
	10	100	34	810
39	1	100	2	98
	2	100	3	195
	3	100	6	289
	4	100	8	381
	5	100	10	471
	6	100	18	553
	7	100	30	623
	8	100	24	699
	9	100	38	761
	10	100	40	821
40	1	100	2	98
	2	100	4	194
	3	100	11	283
	4	100	15	368
	5	100	15	453
	6	100	18	535
	7	100	26	609
	8	100	34	675
	9	100	30	745
	10	100	48	797

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
41	1	100	2	98
	2	100	6	192
	3	100	8	284
	4	100	11	373
	5	100	28	445
	6	100	18	527
	7	100	25	602
	8	100	34	668
	9	100	35	733
	10	100	42	791
42	1	100	0	100
	2	100	2	198
	3	100	7	291
	4	100	15	376
	5	100	15	461
	6	100	26	535
	7	100	34	601
	8	100	24	677
	9	100	42	735
	10	100	42	793
43	1	100	1	99
	2	100	6	193
	3	100	2	291
	4	100	15	376
	5	100	16	460
	6	100	29	531
	7	100	24	607
	8	100	32	675
	9	100	38	737
	10	100	40	797
44	1	100	0	100
	2	100	6	194
	3	100	10	284
	4	100	7	377
	5	100	14	463
	6	100	19	544
	7	100	25	619
	8	100	33	686
	9	100	31	755
	10	100	35	820

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
45	1	100	1	99
	2	100	6	193
	3	100	11	282
	4	100	15	367
	5	100	14	453
	6	100	27	526
	7	100	19	607
	8	100	24	683
	9	100	40	743
	10	100	55	788
46	1	100	3	97
	2	100	7	190
	3	100	4	286
	4	100	11	375
	5	100	19	456
	6	100	17	539
	7	100	26	613
	8	100	26	687
	9	100	30	757
	10	100	42	815
47	1	100	3	97
	2	100	3	194
	3	100	10	284
	4	100	15	369
	5	100	18	451
	6	100	18	533
	7	100	23	610
	8	100	32	678
	9	100	37	741
	10	100	46	795
48	1	100	0	100
	2	100	5	195
	3	100	4	291
	4	100	13	378
	5	100	16	462
	6	100	21	541
	7	100	29	612
	8	100	33	679
	9	100	34	745
	10	100	42	803

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
49	1	100	2	98
	2	100	3	195
	3	100	11	284
	4	100	12	372
	5	100	22	450
	6	100	20	530
	7	100	26	604
	8	100	36	668
	9	100	39	729
	10	100	29	800
50	1	100	2	98
	2	100	6	192
	3	100	9	283
	4	100	7	376
	5	100	13	463
	6	100	24	539
	7	100	14	625
	8	100	35	690
	9	100	43	747
	10	100	36	811
51	1	100	4	96
	2	100	8	188
	3	100	9	279
	4	100	6	373
	5	100	9	464
	6	100	19	545
	7	100	28	617
	8	100	21	696
	9	100	39	757
	10	100	46	811
52	1	100	2	98
	2	100	4	194
	3	100	8	286
	4	100	9	377
	5	100	17	460
	6	100	16	544
	7	100	24	620
	8	100	45	675
	9	100	42	733
	10	100	45	788

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
53	1	100	3	97
	2	100	5	192
	3	100	11	281
	4	100	17	364
	5	100	14	450
	6	100	17	533
	7	100	21	612
	8	100	35	677
	9	100	27	750
	10	100	43	807
54	1	100	0	100
	2	100	3	197
	3	100	6	291
	4	100	11	380
	5	100	12	468
	6	100	22	546
	7	100	17	629
	8	100	32	697
	9	100	41	756
	10	100	38	818
55	1	100	1	99
	2	100	6	193
	3	100	10	283
	4	100	14	369
	5	100	18	451
	6	100	16	535
	7	100	30	605
	8	100	26	679
	9	100	37	742
	10	100	53	789
56	1	100	0	100
	2	100	2	198
	3	100	5	293
	4	100	17	376
	5	100	14	462
	6	100	15	547
	7	100	26	621
	8	100	23	698
	9	100	35	763
	10	100	45	818

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
57	1	100	0	100
	2	100	8	192
	3	100	5	287
	4	100	9	378
	5	100	9	469
	6	100	14	555
	7	100	31	624
	8	100	30	694
	9	100	46	748
	10	100	45	803
58	1	100	0	100
	2	100	4	196
	3	100	4	292
	4	100	9	383
	5	100	16	467
	6	100	16	551
	7	100	26	625
	8	100	40	685
	9	100	40	745
	10	100	53	792
59	1	100	3	97
	2	100	5	192
	3	100	10	282
	4	100	10	372
	5	100	8	464
	6	100	22	542
	7	100	26	616
	8	100	30	686
	9	100	31	755
	10	100	39	816
60	1	100	0	100
	2	100	4	196
	3	100	4	292
	4	100	11	381
	5	100	17	464
	6	100	19	545
	7	100	18	627
	8	100	32	695
	9	100	35	760
	10	100	39	821

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
61	1	100	1	99
	2	100	4	195
	3	100	6	289
	4	100	13	376
	5	100	7	469
	6	100	16	553
	7	100	27	629
	8	100	30	696
	9	100	32	764
	10	100	45	819
62	1	100	0	100
	2	100	5	195
	3	100	7	288
	4	100	8	380
	5	100	17	463
	6	100	20	543
	7	100	32	611
	8	100	24	687
	9	100	45	742
	10	100	38	804
63	1	100	1	99
	2	100	3	196
	3	100	13	283
	4	100	9	374
	5	100	13	461
	6	100	15	546
	7	100	22	624
	8	100	31	693
	9	100	32	761
	10	100	40	821
64	1	100	2	98
	2	100	2	196
	3	100	5	291
	4	100	11	380
	5	100	8	472
	6	100	13	559
	7	100	30	629
	8	100	26	703
	9	100	32	771
	10	100	46	825

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
65	1	100	0	100
	2	100	3	197
	3	100	9	288
	4	100	10	378
	5	100	15	463
	6	100	17	546
	7	100	21	625
	8	100	26	699
	9	100	39	760
	10	100	44	816
66	1	100	1	99
	2	100	3	196
	3	100	9	287
	4	100	17	370
	5	100	12	458
	6	100	24	534
	7	100	23	611
	8	100	39	672
	9	100	31	741
	10	100	47	794
67	1	100	0	100
	2	100	2	198
	3	100	7	291
	4	100	5	386
	5	100	20	466
	6	100	21	545
	7	100	20	625
	8	100	27	698
	9	100	44	754
	10	100	45	809
68	1	100	2	98
	2	100	3	195
	3	100	11	284
	4	100	16	368
	5	100	15	453
	6	100	16	537
	7	100	29	608
	8	100	23	685
	9	100	33	752
	10	100	36	816

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
69	1	100	1	99
	2	100	3	196
	3	100	10	286
	4	100	14	372
	5	100	12	460
	6	100	23	537
	7	100	22	615
	8	100	26	689
	9	100	38	751
	10	100	47	804
70	1	100	1	99
	2	100	5	194
	3	100	9	285
	4	100	15	370
	5	100	9	461
	6	100	32	529
	7	100	10	610
	8	100	28	682
	9	100	49	733
	10	100	45	788
71	1	100	0	100
	2	100	4	196
	3	100	5	291
	4	100	12	379
	5	100	15	464
	6	100	19	545
	7	100	26	619
	8	100	35	684
	9	100	42	742
	10	100	48	794
72	1	100	0	100
	2	100	5	195
	3	100	6	289
	4	100	9	380
	5	100	18	462
	6	100	19	543
	7	100	32	611
	8	100	31	680
	9	100	33	747
	10	100	37	810

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
73	1	100	0	100
	2	100	6	194
	3	100	5	289
	4	100	11	378
	5	100	14	464
	6	100	16	548
	7	100	33	615
	8	100	31	684
	9	100	26	758
	10	100	42	816
74	1	100	1	99
	2	100	5	194
	3	100	11	283
	4	100	12	371
	5	100	14	457
	6	100	13	544
	7	100	22	622
	8	100	36	686
	9	100	28	758
	10	100	37	821
75	1	100	1	99
	2	100	2	197
	3	100	12	285
	4	100	8	377
	5	100	16	461
	6	100	18	543
	7	100	22	621
	8	100	33	688
	9	100	36	752
	10	100	52	800
76	1	100	2	98
	2	100	5	193
	3	100	5	288
	4	100	8	380
	5	100	15	465
	6	100	15	550
	7	100	29	621
	8	100	19	702
	9	100	39	763
	10	100	38	825

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
77	1	100	1	99
	2	100	2	197
	3	100	11	286
	4	100	12	374
	5	100	13	461
	6	100	20	541
	7	100	29	612
	8	100	28	684
	9	100	38	746
	10	100	45	801
78	1	100	0	100
	2	100	1	199
	3	100	7	292
	4	100	10	382
	5	100	13	469
	6	100	25	544
	7	100	22	622
	8	100	32	690
	9	100	34	756
	10	100	45	811
79	1	100	2	98
	2	100	3	195
	3	100	10	285
	4	100	10	375
	5	100	22	453
	6	100	20	533
	7	100	20	613
	8	100	34	679
	9	100	39	740
	10	100	55	785
80	1	100	1	99
	2	100	3	196
	3	100	10	286
	4	100	14	372
	5	100	10	462
	6	100	25	537
	7	100	21	616
	8	100	20	696
	9	100	37	759
	10	100	52	807

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
81	1	100	2	98
	2	100	3	195
	3	100	7	288
	4	100	16	372
	5	100	14	458
	6	100	21	537
	7	100	18	619
	8	100	34	685
	9	100	39	746
	10	100	44	802
82	1	100	4	96
	2	100	6	190
	3	100	7	283
	4	100	18	365
	5	100	11	454
	6	100	26	528
	7	100	24	604
	8	100	24	680
	9	100	40	740
	10	100	43	797
83	1	100	1	99
	2	100	3	196
	3	100	4	292
	4	100	13	379
	5	100	18	461
	6	100	28	533
	7	100	23	610
	8	100	25	685
	9	100	33	752
	10	100	61	791
84	1	100	0	100
	2	100	6	194
	3	100	9	285
	4	100	13	372
	5	100	13	459
	6	100	16	543
	7	100	23	620
	8	100	30	690
	9	100	25	765
	10	100	40	825

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
85	1	100	1	9
	2	100	1	198
	3	100	9	289
	4	100	11	378
	5	100	8	470
	6	100	25	545
	7	100	29	616
	8	100	30	686
	9	100	41	745
	10	100	52	793
86	1	100	2	98
	2	100	2	196
	3	100	5	291
	4	100	10	381
	5	100	9	472
	6	100	23	549
	7	100	28	621
	8	100	34	687
	9	100	36	751
	10	100	54	797
87	1	100	0	100
	2	100	4	196
	3	100	4	292
	4	100	6	386
	5	100	13	473
	6	100	33	540
	7	100	27	613
	8	100	30	683
	9	100	39	744
	10	100	56	788
88	1	100	0	100
	2	100	4	196
	3	100	8	288
	4	100	16	372
	5	100	20	452
	6	100	30	522
	7	100	17	605
	8	100	35	670
	9	100	35	735
	10	100	41	794

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
89	1	100	2	98
	2	100	5	193
	3	100	7	286
	4	100	14	372
	5	100	12	460
	6	100	24	536
	7	100	33	603
	8	100	23	680
	9	100	34	746
	10	100	38	808
90	1	100	1	99
	2	100	6	193
	3	100	7	286
	4	100	6	380
	5	100	15	465
	6	100	19	546
	7	100	25	621
	8	100	32	689
	9	100	35	754
	10	100	43	811
91	1	100	0	100
	2	100	3	197
	3	100	9	288
	4	100	7	381
	5	100	18	463
	6	10	20	543
	7	100	22	621
	8	100	33	688
	9	100	46	742
	10	100	46	796
92	1	100	1	99
	2	100	5	194
	3	100	8	286
	4	100	10	376
	5	100	24	452
	6	100	17	535
	7	100	29	606
	8	100	32	674
	9	100	33	741
	10	100	42	799

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
93	1	100	1	99
	2	100	7	192
	3	100	6	286
	4	100	11	375
	5	100	15	460
	6	100	22	538
	7	100	21	617
	8	100	30	687
	9	100	38	749
	10	100	42	807
94	1	100	1	99
	2	100	5	194
	3	100	15	279
	4	100	7	372
	5	100	14	458
	6	100	19	539
	7	100	30	609
	8	100	25	684
	9	100	32	752
	10	100	52	800
95	1	100	0	100
	2	100	8	192
	3	100	10	282
	4	100	11	371
	5	100	10	461
	6	100	17	544
	7	100	19	625
	8	100	28	697
	9	100	31	766
	10	100	41	825
96	1	100	1	99
	2	100	9	190
	3	100	8	282
	4	100	11	371
	5	100	14	457
	6	100	9	548
	7	100	23	625
	8	100	27	698
	9	100	33	765
	10	100	41	824

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
97	1	100	0	100
	2	100	9	191
	3	100	7	284
	4	100	9	375
	5	100	13	462
	6	100	15	547
	7	100	25	622
	8	100	32	690
	9	100	34	756
	10	100	38	818
98	1	100	1	99
	2	100	6	193
	3	100	10	283
	4	100	14	369
	5	100	16	453
	6	100	16	537
	7	100	29	608
	8	100	24	684
	9	100	44	740
	10	100	45	795
99	1	100	2	98
	2	100	4	194
	3	100	10	284
	4	100	7	377
	5	100	17	460
	6	100	17	543
	7	100	22	621
	8	100	28	693
	9	100	36	757
	10	100	38	819
100	1	100	0	100
	2	100	4	196
	3	100	13	283
	4	100	10	373
	5	100	17	456
	6	100	18	538
	7	100	23	615
	8	100	27	688
	9	100	37	751
	10	100	34	817

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
101	1	1,000	3	997
	2	11	11	997
	3	31	31	997
	4	55	55	997
	5	65	65	997
	6	81	81	997
	7	92	93	996
	8	78	78	996
	9	84	85	995
	10	91	91	995
	11	79	79	995
	12	92	92	995
	13	90	90	995
	14	109	110	994
	15	125	125	994
102	1	1,000	5	995
	2	15	16	994
	3	24	24	994
	4	51	51	994
	5	54	54	994
	6	79	80	993
	7	99	99	993
	8	69	69	993
	9	94	94	993
	10	74	74	993
	11	111	112	992
	12	92	92	992
	13	94	94	992
	14	88	88	992
	15	98	98	992

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
103	1	1,000	3	997
	2	5	5	997
	3	34	34	997
	4	45	45	997
	5	51	51	997
	6	68	68	997
	7	83	83	997
	8	63	63	997
	9	88	88	997
	10	95	95	997
	11	89	90	996
	12	94	94	996
	13	93	94	995
	14	103	104	994
	15	120	120	994
104	1	1,000	3	997
	2	17	17	997
	3	29	30	996
	4	34	34	996
	5	57	57	996
	6	63	63	996
	7	86	87	995
	8	72	72	995
	9	92	93	994
	10	106	106	994
	11	97	97	994
	12	104	104	994
	13	96	98	992
	14	91	91	992
	15	115	115	992

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
105	1	1,000	3	997
	2	19	19	997
	3	32	33	996
	4	51	51	996
	5	70	70	996
	6	81	82	995
	7	87	87	995
	8	89	89	995
	9	76	76	995
	10	91	92	994
	11	96	97	993
	12	106	106	993
	13	92	93	992
	14	102	102	992
	15	110	112	990
106	1	1,000	5	995
	2	14	14	995
	3	19	19	995
	4	46	46	995
	5	58	58	995
	6	74	74	995
	7	75	75	995
	8	81	81	995
	9	91	92	994
	10	86	86	994
	11	99	100	993
	12	97	98	992
	13	82	82	992
	14	111	113	990
	15	107	107	990

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
107	1	1,000	3	997
	2	17	17	997
	3	22	22	997
	4	46	47	996
	5	53	54	995
	6	91	92	994
	7	93	93	994
	8	91	92	993
	9	87	87	993
	10	81	81	993
	11	88	88	993
	12	104	104	993
	13	100	100	993
	14	85	86	992
	15	117	118	991
108	1	1,000	3	997
	2	16	16	997
	3	24	24	997
	4	37	37	997
	5	65	65	997
	6	78	79	996
	7	70	70	996
	8	98	99	995
	9	95	95	995
	10	77	77	995
	11	79	79	995
	12	97	97	995
	13	114	115	994
	14	110	110	994
	15	125	125	994

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
109	1	1,000	2	998
	2	13	13	998
	3	23	23	998
	4	33	33	998
	5	73	73	998
	6	82	82	998
	7	78	78	998
	8	87	87	998
	9	98	99	997
	10	100	101	996
	11	92	93	995
	12	109	110	994
	13	83	83	994
	14	99	99	994
	15	85	85	994
110	1	1,000	1	999
	2	13	13	999
	3	23	23	999
	4	54	54	999
	5	51	51	999
	6	80	80	999
	7	94	94	999
	8	88	88	999
	9	85	85	999
	10	109	109	999
	11	92	92	999
	12	88	88	999
	13	92	94	997
	14	94	94	997
	15	83	84	996

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
111	1	1,000	3	997
	2	16	16	997
	3	24	24	997
	4	47	47	997
	5	58	59	996
	6	79	79	996
	7	77	77	996
	8	78	78	996
	9	89	90	995
	10	91	92	994
	11	92	95	991
	12	105	105	991
	13	102	102	991
	14	106	106	991
	15	116	116	991
112	1	1,000	2	998
	2	11	11	998
	3	24	24	998
	4	41	41	998
	5	51	51	998
	6	80	80	998
	7	77	78	997
	8	100	101	996
	9	83	84	995
	10	88	89	994
	11	92	92	994
	12	105	105	994
	13	93	93	994
	14	91	91	994
	15	107	108	993

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
113	1	1,000	9	991
	2	13	13	991
	3	29	30	990
	4	49	50	989
	5	52	52	989
	6	79	79	989
	7	79	79	989
	8	85	85	989
	9	92	92	989
	10	82	82	989
	11	102	103	988
	12	73	73	988
	13	99	100	987
	14	92	94	985
	15	93	93	985
114	1	1,000	5	995
	2	10	10	995
	3	28	28	995
	4	40	42	993
	5	69	69	993
	6	82	83	992
	7	71	71	992
	8	98	98	992
	9	89	89	992
	10	96	96	992
	11	97	97	992
	12	84	84	992
	13	96	97	991
	14	100	100	991
	15	101	101	991

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
115	1	1,000	2	998
	2	11	11	998
	3	39	39	998
	4	43	43	998
	5	61	61	998
	6	75	75	998
	7	70	71	997
	8	82	82	997
	9	106	106	997
	10	91	91	997
	11	102	103	996
	12	99	99	996
	13	102	102	996
	14	102	103	995
	15	100	100	995
116	1	1,000	6	994
	2	17	17	994
	3	21	21	994
	4	43	43	994
	5	57	57	994
	6	74	74	994
	7	78	78	994
	8	106	107	993
	9	83	83	993
	10	93	94	992
	11	106	107	991
	12	97	97	991
	13	104	105	990
	14	84	84	990
	15	101	101	990

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
117	1	1,000	4	996
	2	20	20	996
	3	26	26	996
	4	42	42	996
	5	66	66	996
	6	63	64	995
	7	78	78	995
	8	88	88	995
	9	101	101	995
	10	87	87	995
	11	84	84	995
	12	105	105	995
	13	106	106	995
	14	106	106	995
	15	90	91	994
118	1	1,000	3	997
	2	18	18	997
	3	25	25	997
	4	45	45	997
	5	62	62	997
	6	88	88	997
	7	88	88	997
	8	82	82	997
	9	83	83	997
	10	85	85	997
	11	101	101	997
	12	93	93	997
	13	96	97	996
	14	92	92	996
	15	115	115	996

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
119	1	1,000	3	997
	2	17	17	997
	3	27	27	997
	4	47	48	996
	5	65	65	996
	6	89	89	996
	7	77	77	996
	8	90	90	996
	9	79	81	994
	10	89	89	994
	11	89	90	993
	12	97	99	991
	13	103	103	991
	14	98	98	991
	15	91	91	991
120	1	1,000	3	997
	2	15	15	997
	3	26	26	997
	4	33	33	997
	5	80	80	997
	6	68	68	997
	7	86	87	996
	8	84	85	995
	9	82	82	995
	10	105	105	995
	11	112	112	995
	12	100	101	994
	13	118	119	993
	14	103	103	993
	15	102	102	993

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
121	1	1,000	4	996
	2	16	16	996
	3	33	33	996
	4	32	32	996
	5	62	64	994
	6	72	72	994
	7	69	70	993
	8	87	87	993
	9	95	95	993
	10	85	85	993
	11	98	98	993
	12	100	100	993
	13	96	97	992
	14	110	110	992
	15	103	103	992
122	1	1,000	5	995
	2	12	12	995
	3	36	36	995
	4	54	54	995
	5	53	53	995
	6	67	67	995
	7	82	82	995
	8	79	79	995
	9	96	97	994
	10	79	79	994
	11	103	104	993
	12	98	98	993
	13	94	95	992
	14	101	103	990
	15	101	101	990

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
123	1	1,000	6	994
	2	16	16	994
	3	25	25	994
	4	50	50	994
	5	61	61	994
	6	74	75	993
	7	80	80	993
	8	75	75	993
	9	81	82	992
	10	96	98	990
	11	96	96	990
	12	103	103	990
	13	104	104	990
	14	100	100	990
	15	98	98	990
124	1	1,000	2	998
	2	11	11	998
	3	22	22	998
	4	51	51	998
	5	60	60	998
	6	60	60	998
	7	85	85	998
	8	81	81	998
	9	103	103	998
	10	90	90	998
	11	124	124	998
	12	94	94	998
	13	89	89	998
	14	100	100	998
	15	105	105	998

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
125	1	1,000	3	997
	2	9	9	997
	3	26	28	995
	4	44	44	995
	5	69	69	995
	6	85	85	995
	7	76	76	995
	8	79	79	995
	9	74	74	995
	10	88	88	995
	11	88	88	995
	12	85	86	994
	13	101	101	994
	14	105	105	994
	15	104	104	994
126	1	1,000	1	999
	2	7	7	999
	3	38	38	999
	4	44	44	999
	5	53	53	999
	6	61	61	999
	7	77	77	999
	8	82	84	997
	9	101	102	996
	10	100	100	996
	11	91	91	996
	12	90	91	995
	13	84	84	995
	14	84	84	995
	15	102	102	995

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
127	1	1,000	6	994
	2	17	17	994
	3	28	28	994
	4	46	47	993
	5	64	64	993
	6	77	77	993
	7	99	99	993
	8	80	80	993
	9	104	104	993
	10	81	81	993
	11	102	102	993
	12	90	90	993
	13	105	105	993
	14	109	109	993
	15	101	101	993
128	1	1,000	2	998
	2	13	13	998
	3	22	23	997
	4	43	43	997
	5	51	51	997
	6	87	88	996
	7	73	73	996
	8	95	95	996
	9	89	89	996
	10	95	96	995
	11	81	82	994
	12	100	100	994
	13	92	92	994
	14	80	80	994
	15	102	102	994

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
129	1	1,000	4	996
	2	18	18	996
	3	37	37	996
	4	44	44	996
	5	53	53	996
	6	78	79	995
	7	75	75	995
	8	87	88	994
	9	84	84	994
	10	101	101	994
	11	77	77	994
	12	87	87	994
	13	97	98	993
	14	104	104	993
	15	107	107	993
130	1	1,000	6	994
	2	10	10	994
	3	23	23	994
	4	34	34	994
	5	56	56	994
	6	73	73	994
	7	76	76	994
	8	70	70	994
	9	76	76	994
	10	91	91	994
	11	107	108	993
	12	94	94	993
	13	106	106	993
	14	91	91	993
	15	96	96	993

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
131	1	1,000	5	995
	2	12	12	995
	3	20	21	994
	4	40	40	994
	5	50	50	994
	6	72	73	993
	7	91	91	993
	8	75	75	993
	9	87	87	993
	10	92	92	993
	11	95	95	993
	12	93	93	993
	13	103	103	993
	14	112	112	993
	15	110	110	993
132	1	1,000	5	995
	2	10	10	995
	3	40	40	995
	4	36	36	995
	5	60	60	995
	6	83	83	995
	7	77	78	994
	8	91	91	994
	9	87	88	993
	10	95	98	990
	11	103	104	989
	12	101	101	989
	13	104	104	989
	14	107	107	989
	15	117	117	989

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
133	1	1,000	10	990
	2	14	14	990
	3	32	32	990
	4	40	40	990
	5	48	48	990
	6	69	69	990
	7	80	80	990
	8	98	98	990
	9	84	84	990
	10	98	98	990
	11	96	96	990
	12	123	123	990
	13	110	110	990
	14	101	101	990
	15	101	101	990
134	1	1,000	9	991
	2	13	13	991
	3	25	25	991
	4	55	55	991
	5	66	67	990
	6	69	70	989
	7	75	75	989
	8	95	96	988
	9	85	85	988
	10	80	80	988
	11	86	86	988
	12	93	93	988
	13	92	92	988
	14	116	117	987
	15	93	93	987

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
135	1	1,000	3	997
	2	14	14	997
	3	24	25	996
	4	50	50	996
	5	67	67	996
	6	69	70	995
	7	84	85	994
	8	79	79	994
	9	98	98	994
	10	85	85	994
	11	94	94	994
	12	98	98	994
	13	104	107	994
	14	119	119	994
	15	93	94	993
136	1	1,000	5	995
	2	14	14	995
	3	41	41	995
	4	49	49	995
	5	51	51	995
	6	74	74	995
	7	84	84	995
	8	82	83	994
	9	106	106	994
	10	81	82	993
	11	93	93	993
	12	94	94	993
	13	96	96	993
	14	110	110	993
	15	93	94	992

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
137	1	1,000	4	996
	2	20	20	996
	3	25	26	995
	4	43	43	995
	5	66	66	995
	6	75	75	995
	7	81	81	995
	8	89	89	995
	9	91	91	995
	10	87	87	995
	11	88	88	995
	12	88	88	995
	13	95	96	994
	14	114	115	993
	15	91	91	993
138	1	1,000	3	997
	2	16	16	997
	3	24	24	997
	4	49	50	996
	5	65	65	996
	6	68	68	996
	7	82	82	996
	8	93	94	995
	9	83	83	995
	10	97	97	995
	11	111	111	995
	12	83	83	995
	13	98	98	995
	14	105	105	995
	15	89	89	995

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
139	1	1,000	3	997
	2	13	14	996
	3	22	22	996
	4	54	54	996
	5	58	58	996
	6	66	66	996
	7	89	90	995
	8	83	83	995
	9	82	82	995
	10	99	100	994
	11	103	103	994
	12	118	119	993
	13	111	112	992
	14	113	113	992
	15	90	91	991
140	1	1,000	6	994
	2	13	13	994
	3	32	32	994
	4	48	48	994
	5	64	64	994
	6	58	58	994
	7	85	86	993
	8	91	91	993
	9	80	81	992
	10	95	95	992
	11	91	91	992
	12	89	89	992
	13	118	118	992
	14	102	103	991
	15	96	96	991

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
141	1	1,000	6	994
	2	15	15	994
	3	27	27	994
	4	49	49	994
	5	59	59	994
	6	88	88	994
	7	80	80	994
	8	90	92	992
	9	93	93	992
	10	106	107	991
	11	87	87	991
	12	89	89	991
	13	100	100	991
	14	103	104	990
	15	116	116	990
142	1	1,000	6	994
	2	22	23	993
	3	18	18	993
	4	37	37	993
	5	63	63	993
	6	62	63	992
	7	89	89	992
	8	76	76	992
	9	91	91	992
	10	96	96	992
	11	90	90	992
	12	103	103	992
	13	98	98	992
	14	92	92	992
	15	108	109	991

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
143	1	1,000	3	997
	2	12	12	997
	3	27	27	997
	4	28	28	997
	5	73	74	996
	6	84	84	996
	7	81	82	995
	8	67	68	994
	9	90	90	994
	10	86	86	994
	11	97	97	994
	12	122	122	994
	13	100	100	994
	14	89	90	993
	15	121	121	993
144	1	1,000	2	998
	2	14	14	998
	3	25	25	998
	4	52	52	998
	5	55	55	998
	6	74	74	998
	7	90	90	998
	8	79	79	998
	9	94	94	998
	10	96	96	998
	11	88	88	998
	12	89	89	998
	13	86	86	998
	14	97	97	998
	15	117	117	998

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
145	1	1,000	3	997
	2	10	10	997
	3	23	23	997
	4	50	50	997
	5	55	55	997
	6	72	72	997
	7	78	78	997
	8	91	92	996
	9	75	75	996
	10	80	81	995
	11	88	88	995
	12	109	110	994
	13	105	105	994
	14	105	105	994
	15	106	107	993
146	1	1,000	6	994
	2	15	15	994
	3	31	31	994
	4	50	50	994
	5	60	60	994
	6	65	66	993
	7	91	91	993
	8	74	75	992
	9	69	69	992
	10	93	93	992
	11	99	100	991
	12	106	106	991
	13	95	95	991
	14	112	112	991
	15	94	94	991

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
147	1	1,000	3	997
	2	11	11	997
	3	27	27	997
	4	30	30	997
	5	58	58	997
	6	70	70	997
	7	96	96	997
	8	78	78	997
	9	88	89	996
	10	89	89	996
	11	108	108	996
	12	102	103	995
	13	104	104	995
	14	104	104	995
	15	111	112	994
148	1	1,000	7	993
	2	11	11	993
	3	14	14	993
	4	52	52	993
	5	64	64	993
	6	80	81	992
	7	85	85	992
	8	86	88	990
	9	90	91	989
	10	99	99	989
	11	94	94	989
	12	121	121	989
	13	100	100	989
	14	95	95	989
	15	106	106	989

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
149	1	1,000	2	998
	2	16	16	998
	3	26	26	998
	4	34	34	998
	5	59	59	998
	6	72	72	998
	7	76	76	998
	8	84	84	998
	9	109	109	998
	10	84	84	998
	11	97	98	997
	12	88	88	997
	13	102	103	996
	14	94	94	996
	15	93	94	995
150	1	1,000	8	992
	2	7	7	992
	3	18	18	992
	4	42	42	992
	5	63	63	992
	6	89	90	991
	7	81	81	991
	8	77	78	990
	9	91	91	990
	10	85	85	990
	11	96	97	989
	12	88	88	989
	13	99	99	989
	14	85	85	989
	15	101	102	988

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
151	1	1,000,000	3,880	996,120
	2	13,860	13,860	996,120
	3	27,320	27,320	996,120
	4	45,210	45,210	996,120
	5	62,170	62,000	996,090
	6	72,790	72,860	996,020
	7	82,150	82,190	995,980
	8	85,980	86,010	995,950
	9	88,800	88,720	996,030
	10	93,190	93,220	996,000
	11	96,680	96,650	996,030
	12	97,060	97,080	996,010
	13	98,990	98,900	996,100
	14	101,010	100,960	996,150
	15	103,420	103,370	996,200
152	1	1,000,000	3,870	996,130
	2	15,130	15,100	996,160
	3	28,320	28,340	996,140
	4	44,530	44,570	996,100
	5	62,030	62,020	996,110
	6	74,560	74,560	996,110
	7	82,000	81,990	996,120
	8	85,890	86,050	996,050
	9	88,740	88,750	996,040
	10	94,870	94,940	995,970
	11	94,850	94,820	996,000
	12	96,640	96,790	995,850
	13	99,610	99,590	995,870
	14	100,630	100,600	995,900
	15	100,790	100,770	995,920

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
153	1	1,000,000	3,670	996,330
	2	14,380	14,380	996,330
	3	28,550	28,580	996,300
	4	44,100	44,080	996,320
	5	61,250	61,270	996,300
	6	73,790	73,860	996,230
	7	81,120	81,070	996,280
	8	87,270	87,270	996,280
	9	89,950	89,930	996,300
	10	92,120	92,100	99,320
	11	94,920	94,190	996,330
	12	97,420	97,380	996,370
	13	99,790	99,890	996,270
	14	100,780	100,730	996,320
	15	102,220	102,170	996,370
154	1	1,000,000	3,560	996,440
	2	14,320	14,320	996,440
	3	27,120	27,130	996,430
	4	44,990	44,960	996,460
	5	61,380	61,430	996,410
	6	74,300	74,250	996,460
	7	81,560	81,580	996,440
	8	85,570	85,630	996,380
	9	89,780	89,700	996,460
	10	93,410	93,440	996,430
	11	95,620	95,610	996,440
	12	98,540	98,540	996,440
	13	99,700	99,660	996,480
	14	101,240	101,210	996,510
	15	103,270	103,270	996,510

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
155	1	1,000,000	3,710	996,290
	2	14,390	14,350	996,330
	3	27,710	27,720	996,320
	4	44,890	44,850	996,360
	5	60,200	60,290	996,270
	6	74,250	74,360	996,160
	7	81,870	81,880	996,150
	8	86,430	86,360	996,220
	9	88,820	88,760	996,280
	10	92,680	92,640	996,320
	11	96,440	96,460	996,300
	12	98,830	98,910	996,220
	13	99,480	99,390	996,310
	14	101,010	100,980	996,340
	15	100,810	100,770	996,380
156	1	1,000,000	3,490	996,510
	2	14,460	14,440	996,530
	3	27,690	27,710	996,510
	4	44,140	44,080	996,570
	5	60,550	60,580	996,540
	6	74,920	74,940	996,520
	7	80,980	80,940	996,560
	8	85,020	85,120	996,460
	9	91,380	91,490	996,350
	10	93,660	93,560	996,450
	11	95,500	95,540	996,410
	12	96,710	96,620	996,500
	13	98,980	98,990	996,490
	14	99,730	99,730	996,490
	15	101,240	101,400	996,330

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
157	1	1,000,000	4,010	995,990
	2	14,520	14,500	996,010
	3	27,500	27,490	996,020
	4	44,790	44,810	996,000
	5	61,790	61,780	996,010
	6	75,010	75,080	995,940
	7	82,380	82,410	995,910
	8	86,520	86,630	995,800
	9	89,080	89,110	995,770
	10	91,430	91,430	995,770
	11	96,770	96,660	995,880
	12	97,600	97,680	995,800
	13	98,990	99,030	995,760
	14	100,520	100,570	995,710
	15	101,850	101,830	995,730
158	1	1,000,000	3,730	996,270
	2	14,890	14,870	996,290
	3	28,260	28,310	996,240
	4	44,410	44,390	996,260
	5	61,320	61,350	996,230
	6	74,670	74,760	996,140
	7	80,590	80,620	996,110
	8	85,780	85,900	995,990
	9	90,340	90,360	995,970
	10	91,150	91,090	996,030
	11	96,010	96,130	995,910
	12	97,730	97,670	995,970
	13	99,480	99,480	995,970
	14	100,700	100,570	996,100
	15	100,540	100,520	996,120

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
159	1	1,000,000	3,800	996,200
	2	14,170	14,240	996,130
	3	28,360	28,440	996,050
	4	44,420	44,440	996,030
	5	61,930	61,910	996,050
	6	74,840	74,860	996,030
	7	82,730	82,710	996,050
	8	86,520	86,490	996,080
	9	87,910	87,910	996,080
	10	92,040	92,040	996,080
	11	95,120	95,150	996,050
	12	98,770	98,870	995,950
	13	100,270	100,220	996,000
	14	101,610	101,860	995,750
	15	100,660	100,640	995,770
160	1	1,000,000	3,630	996,370
	2	14,910	14,920	996,360
	3	28,490	28,560	996,290
	4	44,110	44,050	996,350
	5	61,490	61,530	996,310
	6	74,420	74,470	996,260
	7	82,110	82,110	996,260
	8	84,140	84,170	996,230
	9	90,130	90,120	996,240
	10	91,520	91,510	996,250
	11	94,150	94,190	996,210
	12	98,860	98,890	996,180
	13	99,490	99,430	996,240
	14	100,100	100,020	996,320
	15	102,210	102,250	996,280

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
161	1	1,000,000	3,730	996,270
	2	14,290	14,270	996,290
	3	28,050	28,050	996,290
	4	44,910	44,930	996,270
	5	60,500	60,500	996,270
	6	74,560	74,620	996,210
	7	81,080	81,070	996,220
	8	86,010	86,010	996,220
	9	89,570	89,530	996,260
	10	93,200	93,220	996,240
	11	96,330	96,380	996,190
	12	99,240	99,110	996,320
	13	100,890	100,850	996,360
	14	101,790	101,790	996,360
	15	100,760	100,720	996,400
162	1	1,000,000	3,720	996,280
	2	14,250	14,260	996,270
	3	28,360	28,420	996,210
	4	45,310	45,330	996,190
	5	59,620	59,620	996,190
	6	74,200	74,180	996,210
	7	81,960	81,990	996,180
	8	85,910	85,770	996,320
	9	88,590	88,540	996,370
	10	93,460	93,480	996,350
	11	96,980	97,130	996,200
	12	98,150	98,070	996,280
	13	97,350	97,350	996,280
	14	101,950	101,940	996,290
	15	101,790	101,810	996,270

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
163	1	1,000,000	3,780	996,220
	2	14,580	14,590	996,210
	3	27,890	27,910	996,190
	4	44,520	44,490	996,220
	5	61,690	61,720	996,190
	6	74,100	74,100	996,190
	7	80,970	80,990	996,170
	8	86,070	86,060	996,180
	9	89,040	89,040	996,180
	10	93,090	93,120	996,150
	11	93,840	93,880	996,110
	12	99,600	99,640	996,070
	13	99,710	99,780	996,000
	14	99,480	99,390	996,090
	15	100,500	100,500	996,090
164	1	1,000,000	3,890	996,110
	2	14,460	14,470	996,100
	3	28,340	28,350	996,090
	4	44,800	44,840	996,050
	5	61,400	61,380	996,070
	6	75,840	85,800	996,110
	7	80,830	80,830	996,110
	8	84,350	84,370	996,090
	9	90,430	90,400	996,120
	10	92,800	92,780	996,140
	11	94,870	94,920	996,090
	12	97,020	97,030	996,080
	13	98,900	98,860	996,120
	14	101,430	101,490	996,060
	15	102,180	102,160	996,080

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
165	1	1,000,000	3,440	996,560
	2	14,760	14,770	996,550
	3	28,050	28,050	996,550
	4	45,150	45,140	996,560
	5	60,650	60,670	996,540
	6	73,540	73,590	996,490
	7	82,150	82,140	996,500
	8	85,280	85,360	996,420
	9	89,440	89,450	996,410
	10	93,120	93,140	996,390
	11	94,840	94,780	996,450
	12	98,400	98,410	996,440
	13	100,560	100,520	996,480
	14	100,600	100,710	996,370
	15	102,630	102,750	996,250
166	1	1,000,000	3,590	996,410
	2	14,370	14,370	996,410
	3	27,270	27,300	996,380
	4	43,510	43,510	996,380
	5	61,660	61,720	996,320
	6	74,640	74,640	996,320
	7	82,330	82,310	996,340
	8	86,440	86,540	996,240
	9	89,140	89,140	996,240
	10	91,770	91,770	996,240
	11	96,910	96,860	996,290
	12	95,580	98,590	996,280
	13	101,960	101,900	996,340
	14	99,930	99,970	996,300
	15	102,020	102,090	996,230

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
167	1	1,000,000	3,690	996,310
	2	14,980	15,020	996,270
	3	27,970	27,960	996,280
	4	45,140	45,080	996,340
	5	61,380	61,270	996,450
	6	74,480	74,500	996,430
	7	83,220	83,120	996,530
	8	83,710	83,750	996,490
	9	89,490	89,460	996,520
	10	92,530	92,530	996,520
	11	94,950	94,870	996,600
	12	98,330	98,330	996,600
	13	100,530	100,640	996,490
	14	100,000	99,940	996,550
	15	103,870	103,850	996,570
168	1	1,000,000	3,430	996,570
	2	14,450	14,490	996,530
	3	28,470	28,540	996,460
	4	43,310	43,320	996,450
	5	61,550	61,540	996,460
	6	74,620	74,600	996,480
	7	82,120	82,060	996,540
	8	86,420	86,490	996,470
	9	88,390	88,420	996,440
	10	92,970	92,960	996,450
	11	96,580	96,500	996,530
	12	98,970	99,060	996,440
	13	99,890	99,950	996,380
	14	102,280	102,400	996,260
	15	101,850	101,900	996,210

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
169	1	1,000,000	3,730	996,270
	2	14,720	14,700	996,290
	3	27,570	27,550	996,310
	4	45,720	45,720	996,310
	5	61,600	61,640	996,270
	6	75,590	75,560	996,300
	7	82,930	83,010	996,220
	8	85,190	85,240	996,170
	9	87,840	87,770	996,240
	10	91,330	91,330	996,240
	11	94,390	94,380	996,250
	12	96,180	96,090	996,340
	13	100,390	100,470	996,260
	14	100,580	100,690	996,150
	15	101,270	101,280	996,140
170	1	1,000,000	3,630	996,370
	2	14,700	14,700	996,370
	3	28,050	28,040	996,380
	4	44,970	45,000	996,350
	5	61,400	61,390	996,360
	6	74,490	74,630	996,220
	7	82,940	82,920	996,240
	8	87,740	87,610	996,370
	9	90,160	90,200	996,310
	10	92,930	92,880	996,360
	11	95,090	95,190	996,260
	12	98,310	98,260	996,310
	13	98,190	98,210	996,290
	14	101,140	101,080	996,350
	15	102,460	102,430	996,380

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
171	1	1,000,000	3,610	996,150
	2	14,360	14,340	996,410
	3	27,180	27,230	996,360
	4	43,410	43,460	996,310
	5	61,980	61,980	996,310
	6	74,770	74,740	996,340
	7	82,200	82,320	996,220
	8	85,420	85,550	996,090
	9	89,960	89,970	996,080
	10	92,890	92,790	996,180
	11	96,400	96,430	99,150
	12	98,490	98,390	996,250
	13	100,880	100,880	996,250
	14	101,780	101,770	996,260
	15	100,510	100,540	996,230
172	1	1,000,000	3,880	996,120
	2	14,490	14,490	996,120
	3	28,630	28,640	996,110
	4	44,790	44,810	996,090
	5	60,090	60,150	996,030
	6	75,120	75,160	995,990
	7	80,450	80,400	996,040
	8	87,510	87,450	996,100
	9	89,410	89,370	996,140
	10	93,910	93,880	996,170
	11	95,230	95,270	996,130
	12	98,680	98,630	996,180
	13	100,750	100,810	996,120
	14	101,750	101,680	996,190
	15	102,180	102,220	996,150

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
173	1	1,000,000	3,980	996,020
	2	14,680	14,640	996,060
	3	27,730	27,690	996,100
	4	43,950	44,020	996,030
	5	59,070	59,130	995,970
	6	74,560	74,550	995,980
	7	81,790	81,810	995,960
	8	86,890	86,950	995,900
	9	88,680	88,710	995,870
	10	93,390	93,430	995,830
	11	94,790	94,840	995,780
	12	100,210	100,270	995,720
	13	99,600	99,680	995,640
	14	100,960	100,880	995,720
	15	100,380	100,350	995,750
174	1	1,000,000	3,600	996,400
	2	15,000	15,000	996,400
	3	27,180	27,150	996,430
	4	45,090	45,120	996,400
	5	61,630	61,640	996,390
	6	76,080	76,070	996,400
	7	82,590	82,710	996,280
	8	86,240	86,170	996,350
	9	88,950	88,940	996,360
	10	92,400	92,340	996,420
	11	94,630	94,610	996,440
	12	98,830	98,830	996,440
	13	99,560	99,550	996,450
	14	100,660	100,680	996,430
	15	102,090	102,030	996,490

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
175	1	1,000,000	3,770	996,230
	2	14,930	14,910	996,250
	3	29,190	29,200	996,240
	4	44,350	44,380	996,210
	5	62,050	62,010	996,250
	6	74,130	74,070	996,310
	7	82,890	82,950	996,250
	8	85,900	85,900	996,250
	9	89,310	89,310	996,250
	10	93,740	93,760	996,230
	11	96,120	96,070	996,280
	12	96,770	96,780	996,270
	13	97,990	98,010	996,250
	14	100,310	100,380	996,180
	15	102,460	102,510	996,130
176	1	1,000,000	3,690	996,310
	2	14,010	13,980	996,340
	3	28,300	28,310	996,330
	4	44,760	44,760	996,330
	5	59,910	59,940	996,300
	6	74,400	74,410	996,290
	7	82,030	81,990	996,330
	8	86,850	86,850	996,330
	9	89,450	89,460	996,320
	10	92,810	92,990	996,140
	11	94,590	94,590	996,140
	12	98,520	98,410	996,250
	13	100,120	100,090	996,280
	14	100,950	100,990	996,240
	15	102,000	101,980	996,260

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
177	1	1,000,000	3,670	996,330
	2	14,940	14,920	996,350
	3	27,900	27,900	996,350
	4	45,280	45,280	996,350
	5	62,210	62,150	996,410
	6	75,300	75,340	996,370
	7	83,440	83,400	996,410
	8	85,930	85,970	996,370
	9	89,190	89,170	996,390
	10	93,200	93,230	996,360
	11	94,420	94,460	996,320
	12	97,730	97,740	996,310
	13	99,900	99,920	996,290
	14	100,200	100,150	996,340
	15	103,370	103,420	996,290
178	1	1,000,000	3,600	996,400
	2	14,310	14,290	996,420
	3	28,150	28,110	996,460
	4	43,130	43,100	996,490
	5	61,000	60,990	996,500
	6	74,650	74,640	996,510
	7	80,000	80,720	996,590
	8	86,650	86,700	996,490
	9	88,950	88,960	996,530
	10	93,850	93,830	996,550
	11	95,390	95,430	996,510
	12	97,620	97,640	996,490
	13	100,420	100,410	996,500
	14	101,650	101,640	996,510
	15	101,460	101,370	996,600

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
179	1	1,000,000	3,550	996,450
	2	15,130	15,120	996,460
	3	27,830	27,840	996,450
	4	44,540	44,520	996,470
	5	61,200	61,250	996,420
	6	74,300	74,180	996,540
	7	82,510	82,520	996,530
	8	87,130	87,090	996,570
	9	90,290	90,300	996,560
	10	93,600	93,610	996,550
	11	95,420	95,390	996,580
	12	98,330	98,470	996,440
	13	99,000	98,970	996,470
	14	102,310	102,310	996,470
	15	102,030	102,100	996,400
180	1	1,000,000	3,570	996,430
	2	14,640	14,610	996,460
	3	27,680	27,670	996,470
	4	44,740	44,700	996,510
	5	60,890	60,870	996,530
	6	75,500	75,390	996,640
	7	81,880	81,890	996,630
	8	86,540	86,530	996,640
	9	89,930	89,850	996,720
	10	93,570	93,500	996,790
	11	95,170	95,190	996,770
	12	97,340	97,430	996,680
	13	98,380	98,310	996,750
	14	101,110	101,010	996,850
	15	100,310	100,400	996,760

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
181	1	1,000,000	3,630	996,370
	2	15,060	15,040	996,390
	3	29,040	92,090	996,340
	4	45,740	45,760	996,320
	5	62,110	62,100	996,330
	6	73,280	73,300	996,310
	7	82,740	82,760	996,290
	8	86,800	86,780	996,310
	9	88,320	88,280	996,350
	10	93,010	92,940	996,420
	11	95,430	95,350	996,500
	12	97,250	97,270	996,480
	13	99,460	99,470	996,470
	14	100,800	100,800	996,470
	15	102,770	102,740	996,500
182	1	1,000,000	3,690	996,310
	2	14,400	14,370	996,340
	3	27,450	27,460	996,330
	4	44,740	44,730	996,340
	5	60,220	60,150	996,410
	6	75,200	75,170	996,440
	7	82,290	82,200	996,530
	8	85,820	85,700	996,650
	9	90,160	90,130	996,680
	10	94,660	94,610	996,730
	11	95,120	95,180	996,670
	12	97,490	97,440	996,720
	13	99,410	99,290	996,840
	14	101,820	101,800	996,860
	15	102,590	102,630	996,820

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
183	1	1,000,000	3,980	996,110
	2	14,160	14,160	996,110
	3	27,940	27,930	996,120
	4	43,400	43,470	996,050
	5	61,020	60,960	996,110
	6	74,410	74,390	996,130
	7	82,120	82,150	996,100
	8	86,390	86,490	996,000
	9	89,760	89,820	995,940
	10	93,630	93,670	995,900
	11	95,390	95,420	995,870
	12	97,670	97,680	995,860
	13	100,640	100,690	995,810
	14	103,060	103,050	995,820
	15	99,490	99,450	995,860
184	1	1,000,000	3,550	996,450
	2	14,390	14,440	996,400
	3	28,190	28,170	996,420
	4	44,100	44,070	996,450
	5	62,880	62,820	996,510
	6	73,840	73,840	996,510
	7	82,610	82,680	996,440
	8	85,950	85,860	996,530
	9	88,950	89,000	996,480
	10	91,110	91,150	996,440
	11	95,280	95,340	996,380
	12	96,770	96,810	996,340
	13	98,080	98,030	996,390
	14	101,960	101,960	996,390
	15	101,550	101,470	996,470

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
185	1	1,000,000	3,530	996,470
	2	14,870	14,870	996,470
	3	28,490	28,480	996,480
	4	44,410	44,470	996,420
	5	61,290	61,280	996,430
	6	75,230	75,320	996,340
	7	83,090	83,090	996,340
	8	86,660	86,660	996,340
	9	89,990	89,960	996,370
	10	92,390	92,450	996,310
	11	97,050	97,160	996,200
	12	98,170	98,240	996,130
	13	98,520	98,520	996,130
	14	100,510	100,500	996,140
	15	100,350	100,360	996,130
186	1	1,000,000	3,360	996,640
	2	14,650	14,620	996,670
	3	27,970	27,970	996,700
	4	45,510	45,530	996,680
	5	62,660	62,640	996,700
	6	74,780	74,780	996,700
	7	82,070	82,050	996,720
	8	85,820	85,890	996,650
	9	89,100	89,200	996,550
	10	92,110	92,270	996,400
	11	96,810	96,800	996,400
	12	96,950	96,960	996,390
	13	99,370	99,330	996,430
	14	100,410	100,380	996,460
	15	100,070	100,130	996,400

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
187	1	1,000,000	3,700	996,300
	2	13,980	12,900	996,290
	3	28,580	28,560	996,310
	4	44,690	44,680	996,320
	5	61,520	61,500	996,340
	6	73,830	73,780	996,390
	7	81,860	81,590	996,410
	8	85,530	85,460	996,380
	9	89,690	89,790	996,380
	10	91,910	91,900	996,390
	11	96,740	96,760	996,370
	12	97,400	97,360	996,410
	13	100,790	100,770	996,430
	14	101,980	102,120	996,290
	15	100,880	101,050	996,120
188	1	1,000,000	3,750	996,250
	2	14,010	13,980	996,280
	3	28,680	28,690	996,270
	4	44,120	44,180	996,210
	5	61,500	61,500	996,210
	6	75,360	75,310	996,260
	7	81,030	81,180	996,110
	8	86,030	86,050	996,090
	9	90,620	90,590	996,120
	10	93,770	93,860	996,030
	11	95,140	95,200	995,970
	12	98,720	98,740	995,950
	13	98,490	98,350	996,090
	14	101,610	101,600	996,100
	15	101,210	101,160	996,150

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
189	1	1,000,000	3,690	996,310
	2	14,360	14,390	996,280
	3	28,240	28,240	996,280
	4	44,860	44,870	996,270
	5	62,190	62,180	996,280
	6	75,690	75,690	996,220
	7	82,080	82,040	996,260
	8	85,550	85,600	996,210
	9	90,990	90,970	996,230
	10	92,940	92,900	996,270
	11	95,360	95,330	996,300
	12	96,630	96,640	996,290
	13	98,190	98,160	996,320
	14	102,300	102,370	996,250
	15	103,230	103,310	996,170
190	1	1,000,000	3,640	996,360
	2	14,400	14,400	996,360
	3	27,830	27,770	996,420
	4	44,430	44,420	996,430
	5	59,550	59,600	996,380
	6	74,690	74,770	996,300
	7	82,300	82,210	996,390
	8	88,060	88,080	996,370
	9	88,340	88,370	996,340
	10	93,810	93,690	996,460
	11	96,480	96,440	996,500
	12	97,410	97,360	996,550
	13	99,440	99,460	996,530
	14	99,420	99,420	996,530
	15	102,670	120,660	996,540

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
191	1	1,000,000	3,420	996,580
	2	14,420	14,410	996,590
	3	27,660	27,620	996,630
	4	46,210	46,260	996,580
	5	59,670	59,680	996,570
	6	72,840	72,900	996,510
	7	81,660	81,750	996,420
	8	85,480	85,480	996,420
	9	89,390	89,360	996,450
	10	92,340	92,270	996,520
	11	95,650	95,670	996,500
	12	98,870	98,790	996,580
	13	99,840	99,760	996,660
	14	101,510	101,580	996,590
	15	101,250	101,220	996,620
192	1	1,000,000	4,060	995,940
	2	14,700	14,710	995,930
	3	27,540	27,550	995,920
	4	45,610	45,640	995,890
	5	60,810	60,850	995,850
	6	75,670	75,650	995,870
	7	81,010	81,050	995,830
	8	85,690	85,810	995,710
	9	90,030	90,010	995,740
	10	94,580	94,520	995,790
	11	95,770	95,840	995,720
	12	97,560	97,500	995,780
	13	99,160	99,180	995,760
	14	98,790	98,790	995,760
	15	101,230	101,300	995,690

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
193	1	1,000,000	3,500	996,500
	2	13,970	13,970	996,500
	3	27,930	27,890	996,540
	4	44,200	44,250	996,490
	5	61,590	61,560	996,520
	6	74,570	74,580	996,510
	7	80,580	80,700	996,390
	8	85,420	85,440	996,370
	9	90,400	90,390	996,380
	10	91,840	91,810	996,410
	11	94,160	94,200	996,370
	12	97,190	97,150	996,410
	13	99,750	99,880	996,280
	14	100,720	100,730	996,270
	15	101,540	101,480	996,330
194	1	1,000,000	3,870	996,130
	2	14,150	14,150	996,130
	3	28,120	28,130	996,120
	4	44,820	44,880	996,060
	5	61,140	61,120	996,080
	6	74,490	74,520	996,050
	7	81,840	81,750	996,140
	8	85,460	85,460	996,140
	9	90,360	90,350	996,150
	10	93,200	93,240	996,110
	11	95,910	95,880	996,140
	12	97,100	97,280	995,960
	13	100,370	100,380	995,950
	14	98,500	98,510	995,940
	15	101,060	101,090	995,910

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
195	1	1,000,000	3,620	996,380
	2	14,350	14,370	996,360
	3	27,570	27,550	996,380
	4	43,590	43,660	996,310
	5	61,800	61,820	996,290
	6	74,360	74,400	996,250
	7	82,000	81,950	996,300
	8	86,200	86,180	996,320
	9	89,550	89,620	996,250
	10	92,940	92,950	996,240
	11	96,420	96,480	996,180
	12	97,800	97,790	996,190
	13	99,010	99,020	996,180
	14	101,090	101,050	996,220
	15	102,260	102,280	996,200
196	1	1,000,000	3,200	996,800
	2	14,480	14,440	996,840
	3	27,120	27,160	996,800
	4	44,390	44,370	996,820
	5	61,700	61,640	996,880
	6	75,940	75,980	996,840
	7	81,450	81,530	996,760
	8	86,950	87,000	996,710
	9	90,070	90,120	996,660
	10	92,820	92,740	996,740
	11	96,290	96,290	996,740
	12	97,980	97,970	996,750
	13	100,170	100,240	996,680
	14	99,550	99,600	996,630
	15	101,750	101,720	996,660

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
197	1	1,000,000	3,650	996,350
	2	14,200	14,200	996,350
	3	28,440	28,430	996,360
	4	43,910	43,960	996,310
	5	62,170	62,140	996,340
	6	73,960	74,000	996,300
	7	81,470	81,540	996,230
	8	86,720	86,710	996,240
	9	88,250	88,290	996,200
	10	93,790	93,810	996,180
	11	96,370	96,390	996,160
	12	97,190	97,180	996,170
	13	98,320	98,370	996,120
	14	100,990	101,030	996,080
	15	99,700	99,750	996,030
198	1	1,000,000	3,570	996,430
	2	14,880	14,880	996,430
	3	29,730	92,340	996,460
	4	43,990	43,980	996,470
	5	62,440	62,440	996,470
	6	75,070	74,970	996,570
	7	82,700	82,660	996,610
	8	86,610	86,620	996,600
	9	89,450	89,430	996,620
	10	92,910	93,010	996,520
	11	96,850	96,740	996,630
	12	98,020	98,020	996,630
	13	99,070	99,160	996,540
	14	100,390	100,310	996,620
	15	101,130	101,240	996,510

Sample	Year	Additions (Middle of year)	Retirements (During year)	Balances (End of year)
199	1	1,000	3,590	996,410
	2	14,750	14,700	996,460
	3	28,270	28,230	996,500
	4	44,880	44,890	996,490
	5	61,350	61,400	996,440
	6	75,210	75,190	996,460
	7	82,900	82,970	996,390
	8	86,880	86,890	996,380
	9	89,200	89,280	996,300
	10	93,690	93,740	996,250
	11	95,360	95,320	996,290
	12	97,460	97,480	996,270
	13	99,040	98,930	996,380
	14	98,870	98,950	996,300
	15	102,660	102,620	996,340
200	1	1,000,000	3,760	996,240
	2	15,050	15,050	996,240
	3	27,770	27,780	996,230
	4	44,930	45,020	996,140
	5	61,820	61,720	996,240
	6	74,580	74,580	996,240
	7	82,600	82,680	996,160
	8	85,510	85,390	996,280
	9	87,700	87,690	996,290
	10	95,110	95,250	996,150
	11	95,700	95,670	996,180
	12	98,100	98,100	996,180
	13	101,810	101,890	996,100
	14	99,890	99,880	996,110
	15	99,710	99,710	996,110